

Erratum

The Erratum itself is choke full of mistakes. As I am preparing a corrected version of the book, I see no point in correcting the erratum. As long as the corrected version is not published (which is planned for the end of 2025 at the latest) If you intend to seriously study the book please contact me and I will provide you with the revised version of the sections you intend to read after you promise that they will be for your eyes only.

IF YOU FIND TYPOS WHICH ARE NOT MENTIONNED HERE PLEASE COMMUNICATE THEM TO ME!!

These typos were mostly found by Percy G. Li during and Jinhyun Jung during their careful reading of the book, and I like to express my gratitude here.

Page 13, line above (1.9) please read “for a distribution Φ and a test function $\xi\dots$ ”

Page 14, Exercise 1.4.1. I am overly optimistic here. I am assuming that the reader knows the following fact from integration theory: If a (reasonable) function f defined on an interval I is such the $\int_I dx f(x)\xi(x) = 0$ for each function ξ which is continuous and with support in I then f is zero a.e. on I .

Page 29, equation (2.9). I should have stressed the simple fact that if the operator A corresponds to the observable \mathcal{O} then the operator A^2 corresponds to the observable \mathcal{O}^2 (with the obvious definition of \mathcal{O}^2).

Page 34, replace Exercise 2.5.12 by the following. Consider the space \mathcal{D} of continuous differentiable functions f on $[0, 1]$ such that $f(0) = f(1)$, Consider the operator A on $L^2([0, 1])$ given by $A(f) = if'$. Thus A is symmetric by Exercise 2.5.4. Prove that if g is in the domain of A^\dagger one may find a continuous function h such that $g = h$ a.e. Hint: Fourier series help. [Note to myself: I have doen this but I need to correct the solution].

Page 35, Exercise 2.5.16. Add the warning: Proving that T is one-to-one requires some skill in measure theory.

Page 37, Exercise 2.5.21. It would be more consistent with the sequel to define $T(f)(x) = f(x - a)$. Change in the solution too.

Page 40, three lines before Section 2.8, “from” should be “form”.

Page 42, line 5 please read ...which is not the case of the “function δ_x ”.

Page 43, second half of line after (2.35) please read instead: “Taking $f = \delta_x$ we have $|f\rangle = |x\rangle$ by (2.26) and (2.35) yields”

Page 46, Exercise 2.9.1 Add a coma after the word “formalism.”

Page 50, fourth line of 2.12. Please note that the subgroup of transformations of the type $\lambda 1$ is normal, so that it makes sense to take a quotient by this subgroup.

Page 51 line 14. Add the following footnote: Indeed, given $a \neq 0$ if f is zero outside an interval of length $|a|/2$ then $\|U(a)(f) - f\|_2 = 2\|f\|_2$.

Page 51, 4 lines before Theorem 2.13.2, after weaker condition, add: Then for $x, y \in \mathcal{H}$ the map $a \mapsto (U(a)(x), y)$ is continuous. Thus...

Page 51, two lines into the proof of Theorem 2.13.2, change into “Since $U(0)$ is unitary...”

Page 58, in definition 2.14.9 second line, replace by “translation by s in the i th direction in physical space.

Page 58, line -8, replace by “translation by s in physical space”

Page 61, below (2.83) add the following: The operator $A(t)$ is called the *time-evolution* of the operator A , the value of t remaining implicit. Add in the index: Time-evolution of an operator.

Page 63, four lines above (2.86) please read ‘ “multiplication by $\exp(itp^2/(2m\hbar))$ ”’.

Page 68, three lines above the proof of Th 2.18.2, please replace by; Not only the previous argument implicitly assumes the existence of an eigenvector φ , it implicitly assumes that this eigenvector belongs to the domain of ...

Page 85, line -6: Please read $U(t) = \exp(-itH)$.

Page 87. line below (3.44), replace the second “where” by “and where”.

Page 88 line 3, add a footnote.”As in finite dimension, the “matrix” of U^\dagger is the conjugate transposed of the matrix of U : $(f_k, U^\dagger(f_\ell)) = (U^\dagger(f_\ell), f_k) = (f_\ell, U(f_k))^* = (f_\ell, e_k)^* = (c_k^\ell)^*$ so that $U^\dagger(f_\ell) = \sum_k (c_k^\ell)^* e_k$.

Page 91, line 5 after “consider the case where” add “as in (3.35)”

Page 94, just after (3.36). Replace the first sentence by: The small boxes $B_{\mathbf{k}}$ of side $2\pi\hbar/L$ centered on the points $\mathbf{k} \in \mathcal{K}$ have disjoint interiors

Page 96, line nine, **a** wrong result.

Page 98, three lines before (3.79) no s at contribution.

Page 105, line 2. add a footnote: Here for $x \in \mathbb{R}$, $x \neq 0$, $\text{sign } x = 1$ if $x > 0$ and $\text{sign } x = -1$ if $x < 0$.

Page 109, add a footnote in Def 4.3.1. We do not claim yet that $B_r \in SO^\uparrow(1, 3)$ this will be proved soon.

Page 112, long paragraph in the middle. End the first sentence after the words “mass m ”, and say : “It is a useful fact that it is the orbit...” Line 4 replace p^* = by $p^* :=$. Later: When $m = 0$, the situation is slightly different; the point $(0, 0, 0, 0)$ is invariant under the action of $SO^\uparrow(1, 3)$, but is not the energy-momentum of anything. All the other points of X_0 represent the energy-momentum of a massless particle. For any observer... rest.

Exercise Show that $X_0 \setminus (0, 0, 0, 0)$ is the orbit under $SO^\uparrow(1, 3)$ of the point $(1, 0, 0, 1)$.

Solution: For any $s > 0$ a pure boost transforms this point in the point $(s, 0, 0, s)$ for $s > 0$ and if $\|\mathbf{x}\| = s$ a suitable rotation transforms this later point into (s, \mathbf{x}) .

Page 115, Exercise 4.4.4. The definition of \mathcal{H} should be the definition of \mathcal{H}' and conversely.

Page 122, top. More explicitly, on the one hand, the unitary transformations corresponding to time translations represent the time-evolution of the system. On the other hand, Lorentz transformations can be thought of as changes of coordinates, which are reflected in the state space by the corresponding unitary transformations.

Page 122, last line: replace the part in italics by “of translation by a in space-time”.

Page 126 line 11, should be i.e. **when**

Page 126, four lines above (4.66) “Translation by s along the i th direction in physical space.”

Page 128. The title of Section 4.10 should be: **The states $|p\rangle$ and $|p\rangle$** . I don't know how to correct the source file here.

Page 130, line -3 in Lemma 8.5.2,

Page 131, line 5, replace “the state $|A(p^*)\rangle$ ” by “the state $U(0, A)|p^*\rangle$ ”.

Page 132, line 4 of 5.2, add a space before the word “Definition”. I don't know how to do that.

Page 134 line -8, add a coma before “and”.

Page 135, equation (5.9). If one requires that $\varphi(f)$ is self-adjoint for $f \in \mathcal{S}$ one must have $\lambda = \bar{\tau}$. This remark is however useless because the property that $\varphi(f)$ is self-adjoint for $f \in \mathcal{S}$ is accidental, and many fundamental fields do not satisfy it.

Page 136, footnote 11. The first claim is nonsense, the field (5.9) always satisfies (5.10) (by the same argument as in the proof of Theorem 5.1.5). However, in the case where $\varphi(f)$ need not be self-adjoint for $f \in \mathcal{S}$, the identity (5.10) is not the proper definition of microcausality, see equation (10.39), and it is true that the field (5.9) satisfies (10.39) only when $|\lambda| = |\tau|$.

Page 137, last line. Put the factor r before dr .

Page 138, (5.20). It should be $\exp(-i\alpha \sinh u)$ on the left.

Page 138, Exercise 5.1.8 (b) Replace by: Prove that the functions $g_\varepsilon(r)$... are uniformly bounded over $\varepsilon > 0$ and converge pointwise...

Page 139, two lines after (5.24) Now, since f and f are causally separated (and in particular have compact support) then h has compact support. If a point a belongs to this support, there exists x in the support of f such that $y = a + x$ belongs to the support of g so that $a^2 < 0$ and the support of h is contained in the set $\{x^2 < 0\}$. As proved...

Pages 146 and 147. In equations (6.1) and (6.4) replace $=$ by $:=$.

Page 149. One time above (6.14), in (6.14) and three times below, replace m by \bar{m} . The first time add the following footnote: Throughout the chapter, m denotes the given mass mass associated to the Klein-Gordon field, whereas \bar{m} denotes another “mass” typically associated to an harmonic oscillator.

Page 153, in (6.25) and twice below, m should be \bar{m} , the mass associated to a harmonic oscillator.

Page 157, two lines below (6.35) and in (6.35), \bar{m} instead of m .

Page 158, \bar{m} rather than m in (6.42) and in (6.43) next page.

Page 161, starting middle of the page, until next page, all the m should be \bar{m} .

Page 163, equation (6.57). This equation is correct, but it is needed only in the case $v = u$. Besides, there is bungled sentence. Instead of “Using integration by parts...” just above (6.57) please read:

The function g_k is an eigenvalue of the Laplacian, of eigenvalue $-\mathbf{k}^2/\hbar^2$, as expressed in (6.7). Using integration by parts

$$\int_B d^3\mathbf{x} \sum_{1 \leq \nu \leq 3} (\partial_\nu u(\mathbf{x}))^2 = - \int_B d^3u(\mathbf{x}) \sum_{1 \leq \nu \leq 3} \frac{\partial^2 u}{(\partial x_\nu)^2}(\mathbf{x}),$$

and that the functions g_k form an orthonormal basis, it is straightforward to formally express the Hamiltonian as... Besides, it might be a good idea to denote the summation index by i since the value is from 1 to 3. The same comment applies to (6.50) and (6.86) below.

Page 171, in (6.78), $(2\pi\hbar)^3$.

Page 173, redo the proof of Theorem 6.11.1 as follows.

Proof By Proposition 5.3.1, $\exp(-itH_B)$ is the time-evolution in \mathcal{B} , so that according to (6.66) the time-evolution of $\sqrt{c} \int d\mathbf{x} \varphi(0, \mathbf{x}) f(\mathbf{x})$ is $\exp(itH_B/\hbar)(A(\mathcal{F}_0(f^*)) + A^\dagger(\mathcal{F}_0(f))) \exp(-itH_B/\hbar)$. Now since $\exp(-itH_B/\hbar)$ is the canonical extension to \mathcal{B} of the time-evolution $\exp(itH/\hbar)$ in \mathcal{H} , by Proposition 3.5.2, for any function $g \in \mathcal{S}^3$ we have $\exp(itH_B/\hbar)A(\mathcal{F}_0(g)) \exp(-itH_B/\hbar) = A(\exp(itH/\hbar)\mathcal{F}_0(g))$ (and a similar result for A^\dagger). Since $\exp(itH/\hbar)$ is just multiplication by $\exp(ictp^0/\hbar)$ one has $\exp(itH/\hbar)\mathcal{F}_0(g)(p) = \exp(ictp^0/\hbar)\mathcal{F}_0(g)(p) = \mathcal{F}_t(g)(p)$ and the result follows by (6.66) again.

Page 173, line -5, there is a factor $(2\pi\hbar)^3$ missing in front of the δ .

Page 174, line 10, “then” instead of “them”.

Page 174. line above (6.84) ..then, remembering that $(f_{\mathbf{k}})_{\mathbf{k}\in\mathcal{K}}$ is an orthonormal basis, and that $f_{-\mathbf{k}} = f_{\mathbf{k}}^*$ we find by a straightforward computation (resembling the computation leading to (6.58)) that ...

Page 174 maybe in (6.84) write $c_{\mathbf{k}}^*c_{\mathbf{k}}$ (and the same for b) instead, if only because the eye is trained to see the c^* first. One should then of course rewrite (6.85) in this direction.

Page 174, last line, and top of page 175. All the expressions such as $a(\mathbf{k})^\dagger$ should be replaced by $a^\dagger(\mathbf{k})$.

Page 175 line 4 add a footnote: and ignoring the fact that there is such a term 1 for each term of an infinite sommation.

Page 175, line 9, factor $(2\pi\hbar)^3$ missing in front of the δ .

Page 176, last line, replace buy ...i.e. $\tilde{\omega}_{\mathbf{k}} = c\sqrt{\mathbf{k}^2/\hbar^2}$ and the eigenvalue $-\mathbf{k}^2/\hbar^2$ corresponds to $-a_{\mathbf{k}}^2$.

Page 186, proof of Lemma 8.1.9 and Exercise 8.1.10. It would be more in line with the mainstream terminology to call a Hermitian operator whose eigenvalues are ≥ 0 a positive *semi-definite* Hermitian operator rather than a positive Hermitian operator. (On the other hand, as defined on page 194, a positive *definite* Hermitian operator has all its eigenvalues > 0 .)

Page 194, Excercise 8.4.4 add to the hint. If this fails, the complete solution is given in the solution of Exercise D.12.3.

page 201. I am grateful to Jinhyun Jung for having pointed out to me that physicists do not define the angular momentum by (8.35) but by the opposite operator, so that (8.35) has to be replaced by

$$J_3(x) = -\lim_{\theta \rightarrow 0} \frac{\hbar}{i\theta} (V(\theta)(x) - x) . \quad (*)$$

The reason for this convention is have the formula $V(\theta) = \exp(-i\theta J_3/\hbar)$ which parallels the formulas (2.72) and (2.75). Note that despite the change of sign between

(8.35) and (*) the result of Exercise 8.8.2 is correct (there was a mistake in the original formulation). In order to bring in line with (*) a number of related definitions (in Exercise 9.7.3) a number of sign changes are required in Sections 9.7 and 9.8. I have listed them below, and I have also provided a revised version of these sections. (Unfortunately, the page numbers are those of my version and do not coincide with those of the printed version.)

Page 201, third line of Exercise 8.8.2, replace $j/2 - k$ by $\hbar(j/2 - k)$.

Page 203, three lines before Exercise 8.9.4, add the following: One says that $SL^+(2, \mathbb{C})$ is a double cover of $O^+(1, 3)$,

Page 203, bottom, add the following:

The next exercise show that there is another way to construct a double cover of $O^+(1, 3)$.

Exercise C (a) Prove that one can define a group $SL^-(2, \mathbb{C})$ by adding an element P'' to $SL(2, \mathbb{C})$ with the following rules: $P''I = P''$, $P''AP'' = -A^{\dagger-1}$ whenever $A \in SL(2, \mathbb{C})$. Prove that the homomorphism κ can be extended to an homomorphism κ' from $SL^-(2, \mathbb{C})$ to $O^+(1, 3)$ by setting $\kappa(P''A) = P\kappa(A)$.

(b) Follow the lines of Exercise 8.9.4 to construct a group isomorphic to $SL^-(2, \mathbb{C})$.

Hint: replace the matrix $\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$ by the matrix $\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$.

(c) Consider the matrix $D = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ so that $D \in SL(2, \mathbb{C})$ and $D = -D^\dagger$. Prove that there is an isomorphism ρ from $SL^-(2, \mathbb{C})$ to $SL^+(2, \mathbb{C})$ such that $\rho(P'') = P'D$ and $\rho(A) = A$ for $A \in SL(2, \mathbb{C})$. (But still, the double covers of $O^+(1, 3)$ provided by $SL^+(1, 3)$ and $SL^-(1, 3)$ are intrinsically different.)

Page 205, before Exercise 8.10.1 add the following exercises.

Exercise A For a subset I of $\{0, 1, 2, 3\}$ set γ_I the identity matrix if $I = \emptyset$ and $\gamma_I = \prod_{\mu \in I} \gamma_\mu$ otherwise, e.g. $\gamma_{\{1,3\}} = \gamma_1\gamma_3$. The goal of this exercise is to prove that the 16 matrices γ_I are linearly independent (so that they form a basis of the set of all 4×4 matrices). Thus we have to prove that if $\sum \alpha_I \gamma_I = 0$ where $\alpha_I \in \mathbb{C}$ and the sum is over $I \subset \{0, 1, 2, 3\}$ then each $\alpha_I = 0$.

(a) Prove that the map $L \mapsto \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} L \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$ is an involution which transforms each γ_μ into $-\gamma_\mu$.

(b) Conclude that it suffices to consider the case of the sums $\sum_{\text{card } I \text{ even}} \alpha_I \gamma_I$ and

$\sum_{\text{card } I \text{ odd}} \alpha_I \gamma_I$ and that the second case follows from the first multiplication to the left by γ_0 .

(c) Another involution is given by $L \mapsto S(P')LS(P')$, and this involution preserves γ_0 and changes the sign of γ_i for $i = 1, 2, 3$. Conclude that it suffices to consider that case of the sums $\sum \alpha_I \gamma_I$ where the sum is over $\text{card } I$ even and $\text{card } I \cap \{1, 2, 3\}$ odd (resp. even).

(d) each of the sums in (c) consists of 4 terms of the type $\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$. Show by direct computation that the four corresponding 2×2 matrices A are independent, concluding the proof.

As a consequence, if a matrix L commutes with all the matrices $\gamma(x)$ it commutes with all matrices and is a multiple of identity.

(e) Prove that the set \mathcal{R} of matrices L of the $\sum_I \alpha_I \gamma_I$ with all $\alpha_I \in \mathbb{R}$ is stable by multiplication (left or right) with the matrices γ_μ . Hint: use the commutation relations (8.5). Consequently, \mathcal{R} is an algebra over the real numbers.

(f) Prove that if a matrix $L \in \mathcal{R}$ commutes with all matrices $\gamma(x)$ it is a *real* multiple of identity.

Exercise B Consider the set \mathcal{U} of 4×4 matrices U which are either of the type $U = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$ or $U = \begin{pmatrix} 0 & B \\ A & 0 \end{pmatrix}$ where $\det A = \det B = \pm 1$. Prove that \mathcal{U} is a group. For such U define $d(U) = \det A$. Prove that d is a group homomorphism from \mathcal{U} to $\{-1, 1\}$.

Exercise D For $u \in \mathbb{R}^{1,3}$ with $u^2 = \pm 1 := \varepsilon$ and $x \in \mathbb{R}^{1,3}$ define

$$\theta_u(x) = -x + 2\varepsilon u(x, u) \quad (aaa).$$

(a) Prove that $\theta_u \in O(1, 3)$.

(b) Prove that θ_{e_0} is the parity operator.

(c) For $1 \leq i \leq 3$ prove that θ_{e_i} changes the signs of the coordinates x^0 and x^j ($j \neq i$) of x .

(d) Deduce from (c) that $\theta_{e_1} \theta_{e_2} \theta_{e_3}$ is time reversal.

(e) Prove that every element of $O(1, 3)$ is a product of elements of the type θ_u for $u \in \mathbb{R}^{1,3}$, $u^2 = \pm 1$. Hint. Using (b) and (d) it suffices to prove this for elements of $SO^\uparrow(1, 3)$. When $u^0 = 0$, $-\theta_u$ is “the reflection through the plane perpendicular to u ”, and every rotation is the product of two such reflections. Next, compute the product $\theta_{u_s} \theta_{u_t}$ where for $s \in \mathbb{R}$, $u_s = (\cosh s, 0, 0, \sinh s)$ and show that recalling the notation (4.21) it is the pure boost B^{s-t} .

(f) Prove that $\det(\theta_u) = -1$. Hint: Observe that if $L \in O(1, 3)$ then $L\theta_u L^{-1} = \theta_{Lu}$.

Solution for (f). The required identity is straightforward, and it implies that $\det \theta_u = \det \theta_{Lu}$ so that it suffices to compute $\det \theta_u$ for $u = e_0$ and $u = e_1$.

Exercise E Consider the set \mathcal{L} of invertible 4×4 complex matrices $L \in \mathcal{U}$ for which there exists $\theta(L) \in O(1, 3)$ such that for all $x \in \mathbb{R}^{1,3}$ we have $\gamma(\theta(L)(x)) = L\gamma(x)L^{-1}$.

- (a) Prove that \mathcal{L} is a group and that the map $L \mapsto \theta(L)$ is a homomorphism.
- (b) For $L \in \mathcal{L}$ define $s(L) = \text{sign}(\theta(L)_0^0)$. Prove that this defines a group homomorphism from L into $\{-1, 1\}$. Hint: review Section 4.1.
- (c) Prove that for $u \in \mathbb{R}^{1,3}$ with $u^2 = \pm 1 := \varepsilon$ we have $\gamma(u) \in \mathcal{L}$ and that $\theta(\gamma(u))$ is given by (aaa).
- (d) Prove that for each u as in (c) we have $d(\gamma(u)) = u^2 = s(\gamma(u))$.
- (e) Consider the group $\text{Pin}(1, 3)$ (footnote here explaining that the name is traditional in physics) generated by the elements $\gamma(u)$ with $u^2 = \pm 1$. Prove that the map θ is a two-to-one homomorphism from $\text{Pin}(1, 3)$ onto $O(1, 3)$.
- (f) Prove that minus the identity belongs to $\text{Pin}(1, 3)$ Hint: compute $\gamma_0\gamma_1\gamma_2\gamma_3$.
- (g) Prove that $\text{Pin}(1, 3) \cap \theta^{-1}(SO^\uparrow(1, 3))$ is the group $S(SL(2, \mathbb{C}))$ of matrices of the form (8.44).
- (h) Prove that $\text{Pin}(1, 3)$ is generated by $\text{Pin}(1, 3) \cap \theta^{-1}(SO^\uparrow(1, 3))$ and the elements $\gamma_0, \gamma_1\gamma_2\gamma_3$.

Elements of solution: (c) We use that $\gamma(u)^{-1} = \varepsilon\gamma(u)$, so that $\gamma(u)\gamma(x)\gamma(u)^{-1} = \varepsilon\gamma(u)\gamma(x)\gamma(u) = -\varepsilon\gamma(u)\gamma(u)\gamma(x) + 2\varepsilon\gamma(u)(x, u) = \gamma(x + 2\varepsilon u(x, u))$. (d) We have $d(\gamma(u)) = \det(M(u)) = u^2$. Recalling that $\theta(L)_0^0 = (\theta(L)(e_0, e_0))$ we have from (aaa) that $s(\gamma(u)) = -1 + 2\varepsilon(u^0)^2$. When $\varepsilon = 1$ this is > 0 because $(u^0)^2 \geq 1$ and when $\varepsilon = -1$ this is negative. This proves that $u^2 = s(\gamma(u))$. (e) It follows from Exercise D, (e) that $\theta(\text{Pin}(1, 3)) = O(1, 3)$. It follows from (e) that $d(L) = s(L)$ for $L \in \text{Pin}(1, 3)$. In particular $d(L) = 1$ when $\theta(L) \in SO^\uparrow(1, 3)$. It follows from Exercise A that when $\theta(L)$ is the identity then L is a multiple of the identity. The only such matrices L in \mathcal{U} with $d(L) = 1$ are plus or minus the identity. This proves that θ is two-to-one. (g). A matrix L in $\text{Pin}(1, 3) \cap \theta^{-1}(SO^\uparrow(1, 3))$ is a product $\gamma(u_1) \dots \gamma(u_k)$. Since $\det \theta(L) = 1$ it follows from Exercise D, (f) that k is even, $k = 2n$. Since $M(u)M(P(u)) = u^2I$, the matrix $\gamma(u)$ is of the type $\begin{pmatrix} 0 & A^{\dagger-1} \\ A & 0 \end{pmatrix}$ if $u^2 = 1$ and of the type $\begin{pmatrix} 0 & -A^{\dagger-1} \\ A & 0 \end{pmatrix}$ if $u^2 = -1$. Since $d(L) = 1$ the number of values of k for which $u_k^2 = -1$ is even. This implies easily that $\text{Pin}(1, 3) \cap \theta^{-1}(SO^\uparrow(1, 3)) \subset S(SL(2, \mathbb{C}))$. That $S(SL(2, \mathbb{C})) \subset \text{Pin}(1, 3)$ follows from the fact that $\theta(\text{Pin}(1, 3)) \supset O(1, 3)$ and that $\text{Pin}(1, 3)$ contains minus the identity. Indeed if $A \in S(SL(2, \mathbb{C}))$ then there exists $B \in \text{Pin}(1, 3)$ with $\theta(B) = \theta(A)$ and $B = \pm A$. (h) The group \mathcal{P} generated by these elements is a subset of $\text{Pin}(1, 3)$ and its image is $O(1, 3)$.

Exercise E' The point of this exercise is to provide another proof that the restriction of θ to $\text{Pin}(1, 3)$ is two-to-one. So we have to prove that if $L = \gamma(u_1) \cdots \gamma(u_n)$ commutes with all the $\gamma(x)$ it is the identity or minus the identity.

(a) Deduce from Exercise A that L is a *real* multiple of identity.

(f) Prove that for $u \in \mathbb{R}^{1,3}$ we have $\det \gamma(u) = (u^2)^2$.

(g) Conclude. Hint: by (f) $\det L = 1$ so that if L is λ times the identity, $\lambda^4 = 1$ and $\lambda = \pm 1$ since λ is real!!.

Exercise F Consider two elements P'' and T'' of \mathcal{L} such that $\theta(P'')$ is parity and $\theta(T'')$ is time reversal. Consider the group \mathcal{S} generated by these elements together with $S(SL(2, \mathbb{C}))$.

(a) Prove that the restriction of θ to \mathcal{S} is two-to-one onto $O(1, 3)$. Hint. Consider the homomorphism \det from \mathcal{L} to $\{-1, 1\}$ given by $\det(L) = \det(\theta(L))$. Show that exists $a, b \in \{0, 1\}$ such that for $L = P''$ or $L = T''$ we have $d(L) = s(L)^a \det(L)^b$. Consequently this relations holds for all $L \in \mathcal{S}$ and a multiple of identity in \mathcal{S} is the identity or minus the identity.

(b) Prove that one can choose P'' and T'' such that $P''^2 = \pm 1$ and $T''^2 = \pm 1$ but that one always have $P''T'' = -T''P''$. Hint. If $L \in \mathcal{L}$ then $iL \in \mathcal{L}$, and if $\theta(L)$ is the identity, L is a multiple of identity.

Page 205, remove Exercise 8.10.1

Page 206, right after Exercise 8.10.1 add the following:

In Exercise F we have constructed four different double covers of $O(1, 3)$. The next exercise provides eight different ways to construct such a double cover of $O(1, 3)$.

Exercise G Consider $\varepsilon_1, \varepsilon_2, \varepsilon_3 = \pm 1$.

(a) Show that one can construct a group $P(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ by adding to $SL(2, \mathbb{C})$ two new elements P', T' with the following rules, for each $A \in SL(2, \mathbb{C})$.

$$P'AP' = \varepsilon_1 A^{\dagger -1}; T'AT' = \varepsilon_2 A; P'T' = T'P'(\varepsilon_3 I)$$

together with (of course) $P'I = P'$ and $T'I = T'$.

(b) Prove that one can extend the homomorphism κ from $SL(2, \mathbb{C})$ to $SO(1, 3)$ into a two-to-one homomorphism κ for $P(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ to $O(1, 3)$ by setting $\kappa(P') = P$ and $\kappa(T') = T$ where T is time reversal in $O(1, 3)$. Prove that this homomorphism is two-to-one.

(c) Prove that the construction of Exercise F (b) “are identical” to $P(\pm 1, \pm 1, -1)$.

Page 206, Exercise 8.10.2, in (c) write: For these representations, construct **all** representations...

Page 206, Exercise 8.10.3, should be moved after Exercise 8.10.4. It should say “If you have succeeded in solving Exercises 8.4.4 and 8.10.2... In the fourth line, $S(A)(P')$ should be instead $S(P')$. In the solutions of the exercise, add the solution Solution: Let us denote by $S(A)$ the action $S(A) = A \otimes A^{\dagger-1}$ of $SL(2, \mathbb{C})$ on $\mathbb{C} \otimes \mathbb{C}$, so that S is the representation $(1, 1)$. Consider the map N which looks as a tensor $y \in \mathbb{C} \otimes \mathbb{C}$ as a 2×2 matrix $N(y)$. It is proved in Proposition D.12.2 that $N(S(A)(y)) = AN(y)A^{*-1}$. Recalling Lemma 8.1.4 and the matrix J there, we have $C^{*-1} = JC^{\dagger}J^{-1}$ so that $N(S(A)(y))J = AN(y)JA^{\dagger}$. To each $x \in \mathbb{C}^4$ we may associate the matrix $M(x)$ by the formula (8.19). Consider then the map W from $C \otimes C$ to \mathbb{C}^4 such that $N(y)J = M(W(y))$. Then $M(WS(A)(y)) = AM(W(y))A^{\dagger}$. The relation $AM(x)A^{\dagger} = M(\kappa(x))$ holds for $x \in \mathbb{R}^{1,3}$ so that it holds for $x \in \mathbb{C}^4$. Thus $M(WS(A)(y)) = M(\kappa(A)W(y))$ and thus $S(A) = W^{-1}\kappa(A)W$, an explicit formula showing that R and κ are equivalent representations. To extend S to $SL^+(2, \mathbb{C})$ in a way which is equivalent to the extension of κ to $SL^+(2, \mathbb{C})$ it then suffices to ask for the relation $S(P') = W^{-1}PW$ where here P denotes the operator on \mathbb{C}^4 which changes the sign of all but the first coordinate. The result follows by explicit computation of the right-hand side.

Page 210, below (9.7). For consistency of notation, for $\varphi \in L^2(X_m, \mathcal{V}, d\lambda_m)$ we denote by $|\varphi\rangle$ the corresponding state, and we are going to prove that

$$U(a, A)|\varphi\rangle = |\tilde{U}(a, A)(\varphi)\rangle. \quad (mb)$$

Recalling the ...

Below, left-hand sides of (9.8) and (9.9), $|\varphi\rangle$ instead of φ .

Page 210, footnote 10, add: As we are not fussy mathematicians, here and below we do not distinguish between functions and classes of functions in L^2 .

Page 211, left hand side of first equation, $U(a, A)|\varphi\rangle$. Two lines below this equation, “this yields (mb)”.

Page 217, Exercise 9.4.3 (a) This also holds for $m = 0$ so maybe one should write instead $D_p^{-1}D_p \in G_{p^*}$ instead.

Page 218, move footnote 25 specifying that we consider only reasonable functions to just above (9.27). [Not needed?]

Page 219, replace the first sentence after Def 9.5.1 by: Thus, the restriction of V to G is simply the restriction of V to G and \mathcal{H}_0 , a representation on \mathcal{H}_0 .

Page 220, equation (9.33). Please read $C \in SL(2, \mathbb{C})$ rather than $C \in SU(2)$. The implication is true for all $C \in SL(2, \mathbb{C})$ and later in the proof it is used that way.

Page 221, line 10 from bottom. ...from X_m to \mathcal{H}_0 for which the quantity (9.28) is finite.

Page 224, Lemma 9.6.3, please read $A \mapsto A^*$.

Page 226, line 8, ...with $\lambda > 0$ so that $\lambda = 1/B_{1,1}$ since $|A_{1,1}| = 1$.

Page 224, Definition 9.6.2. Replace the definition by $\hat{\pi}_j(A) = a^j$ and in the line below remove "The reason for the minus sign will appear later".

Page 225, in (9.42) change the exponent j into $-j$.

Page 226. In the proof of Proposition 9.6.6, change the sign of all the exponents containing a j .

page 226. In the definition of \mathcal{H}_j replace $f(\theta v) = \theta^j f(v)$ by $f(\theta v) = \theta^{-j} f(v)$.

Page 227, line 8, replace $\pi_{0,-j}$ by $\pi_{0,j}$.

Page 227, line 15, at the end of the line write now $V(A)(g) = a^j g = \hat{\pi}_j(A)g$.

Page 227, line 17, replace $\hat{\pi}_{-j}$ by $\hat{\pi}_j$. Line 18, replace $\pi_{0,-j}$ by $\pi_{0,j}$.

Page 227, last line, replace $\pi_{0,-j}$ by $\pi_{0,j}$.

Page 228, first line, replace $\pi_{0,j}$ by $\pi_{0,-j}$.

Page 228, last line of Proposition 9.6.13, replace $\pi_{0,-1}$ by $\pi_{0,1}$.

Page 229, line below (9.55). In the sentence commencing there, say: "The case of $\pi_{m,j}$ corresponds to the choice $V = \pi_j$. Then the overwhelming"

Page 230, line 2, replace $\exp(ij\theta/2)$ by $\exp(-ij\theta/2)$.

Page 230, line 15. Replace the expression $U(0, A)(\varphi)(p) = \exp(ij\theta/2)$ by $U(0, A)(\varphi)(p) = \exp(-ij\theta/2)$.

Page 230, in each of the last two lines before the statement of Exercise 9.7.3 a minus sign has to be added in an exponent.

Page 230 a minus sign has to be inserted in the definition (9.59).

Page 231, a minus sign has to be inserted in the definition (9.60).

Page 231, line 5, replace $\pi_{0,1}$ by $\pi_{0,-1}$.

Page 233, in the first displayed equation replace A by B everywhere. The line below, say “Using (9.66) for $C = D_{B(p^*)}^{-1}B$ and $A = D_{B(p^*)}$.”

Page 233, last two lines, replace twice “has to be zero” by “is zero”.

Page 237 before Theorem 9.10.4 This provides a simple expression for $\|u\|_p$.

Page 237, Theorem 9.10.4 It would be less confusing to give the expression for the square of the norm as in th 9.5.3.

Page 238, line above (9.77), Recalling that $\gamma^\mu = \eta^{\mu\nu}\gamma_\nu$, the quantities $a^\mu = \gamma^\mu$ satisfy (9.76) and the equation....

Page 239, line after (9.78) replace by: where we write $\kappa(A^{-1})(x)$ rather than the short-hand $A^{-1}(x)$.

Page 239, four lines from bottom. Instead of
“It holds that $\gamma_\mu S(A) = S(A)\kappa(A)^\nu_\mu \gamma_\nu = S(A)\gamma_\nu \kappa(A)^\nu_\mu$ ”
please read
“It holds that $\gamma_\mu S(A) = S(A)\kappa(A^{-1})^\nu_\mu \gamma_\nu = S(A)\gamma_\nu \kappa(A^{-1})^\nu_\mu$ ”.

Page 240, Exercise 9.11.3, second line, read “so that $i\hbar\widehat{\partial}_\mu(f)(p) = p_\mu\hat{f}(p)$.”

Page 243 (9.86) It is P rather than P' .

Page 244, first line, middle .. and by Lemma 8,1,4 we have....

Page 245, below def 9.13.3. This representation models the photon. The state space

of the photon.. At the end of the paragraph, add the sentence: It will help to keep in mind that the state space of the photon is a space of functions on X_0 valued in a space of dimension 2.

Page 246, middle, around (9.96). This is not well written, \mathcal{H}_m consists only of the functions φ such that $(\varphi, \varphi)_\eta < \infty$.

Page 245, line 10 from bottom. I am not sure which one is left polarization and which one is right polarization.

Page 246, Exercise 9.13.5. The functions of \mathcal{H}_m are defined only on the mass shell, and the formula (9.97) does not make sense since $A^{-1}(p)$ may not belong to the mass shell when p does, so that $O(1, 3)$ has to be replaced by the subgroup $O(1, 3)^+$ of isochronic transformations (check this).

Page 246, six lines from bottom: ...space of functions φ from X_0 to ... four lines from bottom. There, for $A = P'$ and $x \in \mathbb{C}^4$ $P'(x) = P(x)$ where P is the parity operator, and for $A \in SL(2, \mathbb{C})$

Page 247, line 3, for $A \in SL^+(2, \mathbb{C})$ we have $A(\mathcal{V}_p) \subset \mathcal{V}_p$ so that $A(\mathcal{V}_{A^{-1}(p)}) \subset \mathcal{V}_p$ and thus $V(a, A)(\mathcal{H}) \subset \mathcal{H}$.

Page 247, Equation (9.102). Since Definition 9.6.2 was changed, it seems that here one should exchange x_L and x_R but I have to consider this.

Page 248, middle, proof of (b) middle of line add ..”and from (9.85) that”...

Page 248, line after (9.106), ...the claim that π ...

Page 249, two lines after (9.106), (9.7) does not define $\pi_{0,2}$...

Page 249, three lines after (9.106), π_2 should be $\hat{\pi}_2$

Page 249, second key idea, please read $\kappa(A)(p^*) = p^*$.

Page 255, line 7 from bottom, add a footnote: Keeping in mind that ψ_k^+ is antilinear, we have fomulas such as $\psi_k^+(\xi^*) = \int d^4x \psi_k^+(x)\xi(x)$. for $\xi \in \mathcal{S}$. [does not seem to be the right place].

Page 258. last line, replace by: Since $f_k(0) \in \mathcal{F}$, by (10.10) we have $G_k(C) = V(C)^{-1}G_k(I) = V(C)^{-1}\Pi(e_k)$ and (10.14) follows.

Page 259, last line and line -4, (10.8) should be (10.6).

Page 261, Line 5, replace by the following “An alternative way to look at this formula is to use (remembering that ψ_k^+ is antilinear) that $\psi_k^+(\xi^*) = \cdot$ ”. First line after the third displayed equation, read f_s rather than f_s'' .

Page 261, last line of proof of Prop 10.5.1. Using (10.23) and (10.29), then (10.28) follows from (10.25).

Page 264 line 5, ..which is then...

Page 266. lemma 10.7.4. If the function $f(p)$ is an homogeneous polynomial in the components of p , ...[It is not clear to me what the issue is here. It is p instead of \mathbf{p} ?

Page 271, the notation in the footnotes is not consistent with the notation in the text, it should be $F^{\nu\mu}$ not $F^{\nu,\mu}$, etc.

Page 274, line-3, we set $D_{k,\ell} := \dots$

Page 275, line six, is it really “in the quantities p^μ ? Rather, it should be in the components of \mathbf{p} .

Page 283, look at the following comment: second to last line: π_{-1} should be π_{+1} , and helicity -1/2 should be helicity +1/2. The opposite applies to the next page.

Page 284, in (10.102) and (10.103) the roles of a and b should be exchanged.

Page 285, bottom, part starting with: we consider the vectors, remove this part, and instead say: Considering a vector of \mathbb{C}^4 as a pair of vectors of \mathbb{C}^2 consider the map $W : \mathbb{C}^2 \rightarrow \mathcal{G}$ given by $W(x) = (x, x)$ and the map $W' : \mathbb{C}^2 \rightarrow \mathcal{G}'$ given by $W'(x) = (Jx, -Jx)$ where J is given by (10.65). Then (10.16) and (10.32) hold (recalling that $V' = V$). Denoting by (f_1, f_2) the canonical basis of \mathbb{C}^2 .

Then replace (10.107) by

$$u(\mathbf{p}, s) = S(D_p)W(f_s) ; v(\mathbf{p}, s) = S(D_p)W'(f_s)^* .$$

Last line:

$$\sum_{s \leq 2} W(f_s)W(f_s)^\dagger ; \sum_{s \leq 2} W'(f_s)^*W'(f_s)^{\dagger*} .$$

Page 292, line -2, there is an extraneous period.

Page 293 Middle of 1021, redo from “let us give the formulas” For the case of intrinsic parity 1 we consider $u(\mathbf{p}, s)$ and $v(\mathbf{p}, s)$ given by (10.107) were now $W(x) = (x, x)$ and $W'(x) = (Jx, Jx)$. The field is then given by the formulas (10.110) and (10.11) but with $a^\dagger(\mathbf{p}, s)$ rather than $b^\dagger(\mathbf{p}, s)$ in (10.111), since the anti-particle coincides now with the particle. For the case of intrinsic parity -1 we simply take $W(x) = (x, -x)$ and $W'(x) = (Jx, -Jx)$.

Page 294, after the end of the first paragraph, The reason for this failure is not deep and in fact has nothing to do with the inclusion of parity. Denoting by G_0 the little group of of the pint $(1, 0, 0, 1)$ the representation $\pi_{0,2}$ is induced (in the terminology of of Theorem 9.4.2 by the representation $\widehat{\pi}_2$ of G_0 . Reproducing the analysis of Proposition 10.4.2 to be able to construct the annihilation part of the field (Footnote: the same difficulty occurs for the creation part of the fields) one has to find a subspace \mathcal{G} of \mathbb{C}^4 such that the restriction of $S(= \kappa)$ to \mathcal{G} is equivalent to $\widehat{\pi}_2$.

Exercise Prove that such a space does not exist.

Solution: For a vector $c \in \mathcal{G}$ one must have $S(A)c = c$ whenever A is of the type (9.40) with $a = 1$. It is straightforward that this implies that $c^0 = c^3$ and $c^1 = c^2 = 0$. But then $S(A)c = c$ whenever $A \in G_0$.

To end up this chapter, let us go deeper in the topic of parity. We denote by G_0^+ the little group of $SL^+(2, \mathbb{C})$ corresponding to the point $(1, 0, 0, 1)$.

Page 294, Exercise 10.22.3. Remove part (b) and remove the mention (a) to part (a).

Page 301, equation (11.8) please read

$$(H_0 - \lambda_0 \mathbf{1})w_1 = -(H_I - a_1 \mathbf{1})v_0 = \dots$$

and the line below please read:

\dots on which $H_0 - \lambda_0 \mathbf{1}$ is \dots

Page 301, second display to the last, on the right hand side please read $\sum_{n \geq 1} (H_I v_0, v_n)(v_n, w_1)$. On the next line please read: and since $(H_I v_0, v_n) = (v_n, H_I v_0)^*$ we obtain...

Page 303, last line of exercise 11.1.3 replace by : Explain heuristically why for small g this give the correct result at order g^2 . Sadly, we will see examples where z is not small, and then the method lacks credibility.

Page 305, line below (11.25), please read: Setting $V_0(t) = 1...$

Page 305, in equation (11.27) the last integral should be $\int_0^{\theta_1} d\theta_2 \tilde{H}_1(\theta_2)$.

Page 307, Exercise 11.2.3, the hint is non-nonsensical, $U(t)$ is not a one-parameter group, but the proof of (11.37) carries out just the same.

Page 309, the sentence above (11.44) can be confusing. It simply tries to explain what happens in (11.43). Comparing with (1.13), (11.44) is simply a heuristic reformulation of (11.44).

Page 313, line -5, replace “we will use” by “we consider using”

Page 314, line 6. Replace with: Instead of (5.31) let us consider the following (time independent) function φ on $\mathbb{R}^3...$ Line -8: At some imprecise level the operator $\varphi(\mathbf{x})....$

Page 315, (11.66) and (11.67). Write instead $\exists \mathbf{k}, m_{\mathbf{k}} = n_{\mathbf{k}} - 1, \forall \ell \in \mathcal{K}...$

Page 316, third displayed equation, the last term is $\exp(i\mathbf{p} \cdot \mathbf{x})$.

Page 317, Exercise 11.5.5 ...rate of transition per unit **of** time...

Page 319, lines 6,7,9, the occurrences of n should be replaced by 0.

Page 326, Third line of the proof of Corollary 12.2.2, please read $\varphi = \dots$ instead of $\psi = \dots$. Last line of the proof, read ...which (12,6) holds is closed.

Page 335, last line of footnote ...does not change with time...

Page 326, second line of Exercise 12.2.3. In momentum space, H_0 is multiplication by $\mathbf{p}^2/(2m)$ so that $U_0(T) = \exp(-itH_0)$ is multiplication by $\exp(-it\mathbf{p}^2/(2m))$ (and not as stated by $\exp(it\mathbf{p}^2/(2m))$)

Page 336, beginning of line 12, please read $t \geq 0$ rather than $t \geq T$. (Since ψ is already the time evolution at T of φ .)

Page 337, Theorem 12.4.1. Two lines after (12.32) please read
“... that the function θ occurring there is zero in a neighborhood of the z axis.”

Page 337, footnote 34 is nonsensical. Last line please read: ...the direction \mathbf{p}_0 of the detector is different from the direction of the z -axis.

Page 338, 339. Several occurrences of the expression “the right hand side of (12.39)” have to be replaced by the expression “the quantity (12.40)”.

Page 344, It is nonsense to say that $a_i^\dagger(\mathbf{p})|0\rangle$ is just a notation, see (12.58) on page 345. Starting with the third sentence of the second paragraph, replace the text with the following: A natural continuous basis describing one-particle states consists of the elements $a_i^\dagger(\mathbf{p})|0\rangle$. Here i accounts for the particle type (for simplicity we will consider only spinless particles, otherwise we should also account for the spin of the particle) and for $\mathbf{p} \in \mathbb{R}^3$ the operator $a_i^\dagger(\mathbf{p})$ is defined as in (5.28). The quantity $a_i^\dagger(\mathbf{p})|0\rangle$ is an improper state which makes sense only when integrated against a test function. To describe incoming two-particle states we will similarly use the continuous basis $a_{i_1}^\dagger(\mathbf{p}_1)a_{i_2}^\dagger(\mathbf{p}_2)|0\rangle$ which represents....

Page 344, line above (12.46) please replace $\varphi = |a_1^\dagger(\mathbf{p}_1)a_2^\dagger(\mathbf{p}_2)|0\rangle$ by $\varphi = a_1^\dagger(\mathbf{p}_1)a_2^\dagger(\mathbf{p}_2)|0\rangle$.

Page 345, (12.58) is not an assumption, it is written on top of page 141.

Page 353, line 5. Please read:

and the Hamiltonian H_I is the operator V “multiplication by the function $V(\mathbf{x})$ ”...

Page 356, line 11, Condition (13.21) (parenthesis are missing)

Page 357, Exercise 13.5.1, last line of (a), please read ...is invariant under the representation $S \otimes S$ where $S(C) = T(C^{-1})^T$.

Page 363, equation (13.38). In the first 4 lines, on the right-hand side, ξ has to be replaced by ξ^* . Alternatively, it might be nicer to integrate against the function $\xi(\mathbf{p})^* f(x)$.

Page 364, in the label of Figure 13.1 and a few lines below there is the notation b -particle rather than b particle as is used after that.

Page 365, in equation (13.41), in the right-hand side it should be $\prod_{\{\ell, \ell'\} \in \mathcal{P}}$.

Page 368 line 4, formula (13.44).

Page 380 line 7, one obtains the diagram (c) of Figure 13.4

Page 380, expend footnote 50. Later on, when studying Feynman diagrams, we will find more convenient not to distinguish between internal and final vertices, and to orient each external line from the internal vertex to the corresponding external vertex. In that case we can reformulate the previous rule as follows: for each external line from an internal vertex corresponding to the variable x_j we add a factor $\exp(i(x_j, p))$ where p is the four-momentum flowing from the internal vertex to the corresponding external vertex.

Page 390, bottom, There is the unimportant issue that Σ is defined by giving a minus sign for the moment entering the vertex, which is not compatible with what I do later (but does not change the result since $\delta(p) = \delta(-p)$).

Page 391, line 12. Please read: “We assign a factor $-ig..$ ” (rather than ig)

Page 391, line -4 and -8, please read “ $\delta^{(4)}(\Sigma_v)$ ” rather than $\delta(\Sigma_v)$.

Page 392, please replace the second and third lines of 13.17 by the following:
...to scattering amplitudes that involve *divergent diagrams*, i.e. diagrams whose value is given by a divergent integral.¹ In this section we start the task of renormalizing the scattering amplitude (13.96) in φ^4 theory, computed at order 2 in perturbation theory.

Page 398, line 4. Please observe the important fact that if $a, \epsilon > 0$ the numbers $z = x + iy$ such that $z^2 = a - i\epsilon$ always satisfy $xy < 0$ so that poles at these points do not prevent to use Wick’s rotation.

Page 398, line -11, ..when $w_0 + \sqrt{(\mathbf{p} - \mathbf{w})^2 + m^2} < 0$ because then

Page 399 after line 6, add the following: This is because the function $f(z) = (-z^2 + \mathbf{p}^2 - u(1-u)w^2 + m^2 - i\epsilon)^{-2}$ does not have poles in the first or third quadrants, since the square of a number in one of these quadrants has an imaginary part which is ≥ 0 .

Page 400, in (13.127) there should be a minus sign in the definition of W .

¹Please note the in the definition of a divergent diagram we need not distinguish between contraction and Feynman diagrams as the values of these differ just by a combinatorial factor. Note also that the objective is not necessarily to assign a value to each diagram, but only to certain sums of these diagrams.

page 401, in (13.132) there is a minus sign in front of the last term.

Page 401, rewrite last three lines: We now complete the proof that (13.117) does lead to a physical predilection (assuming that the term $O9g^3$) is small. Our first task is to be more explicit about the value of M .

Page 402, tow lines after (13.137), two **internal** vertices.

Page 402line -6 our prediction (13.117) for the scattering amplitude... Line after (13.140). Please observe that this prediction is Lorentz invariant, as any sensible formula should be.

Page 403, line 2, add a footnote saying the from now on the subscript 2 to the quantity M to indicate that these are the contributions of diagrams with 2 internal vertices. In pages 403 and 404 add a subscript 2 to all the quantities M .

Page 405, below (13.149), suppress the mention “where M_2 is....”

Page 406, just before 13.22. Add the sentence. However, it is certainly easier to get a precise picture on how renormalization works in the case of diagrams with two loops by studying 13.24 rather than the general approach of Part IV.

Page 406, line 14 from bottom, ... c is touched by an internal line. The internal line cannot come from b because there would be only 2 internal lines left out of b , so that there would be an line from a to itself which is forbidden.

Page 407, Figure 13.15. The first diagram on the left is obviously wrong, since each diagram should have two incoming external edges and two outgoing external edges. To get the correct diagrams (there are two of them) rotate the figure 90 degrees, and pick two of the four external edges as incoming edges and two as outgoing edges. There are two cases, depending on whether the lonely external edge is incoming or outgoing.

Page 407, second line from bottom, please read: ...obvious now but will appear...

Page 417, nest to last paragraph: rewrite. as follows. Let us observe that the term D_2 .. works. This is remarkable. One way to understand what happen is that the term $g^2 D_2 = -ig^2 M_2^0$ removes the divergence at the level of the loop. It is in fact designed precisely to remove the divergences of the diagrams with two internal vertices. The important fact is that the same term $g^2 D_2$ work for all possible diagrams

with two internal vertices. Now in the diagrams of the type (2) or (3) of figure 13.15, there are subdivergences in each loop. When contracting such a loop into one single counter-term vertex one obtains a diagram (b) or (c) of figure 13.19. The part of the diagram outside the contracted loop is unchanged, and the contribution of the term $g^2 D$ in the contracted counter-term vertex exactly compensates the divergence of that loop. (This is not surprising, since the same term works for any loop.) So there remains only the overall divergences to take care of, but these are taken care of by the term $g^3 D_3$ in the contribution of the diagram (a) of Figure 13.19. line -2 decomposition (13.71)

Page 418 line 8, the q^* should be p^* . Displayed equation below (13.191) why not write the same formula as in (13.167)?

Page 425, line nine ...according to which points...

Page 426, line -10, ...which without loss of generality we assume to be of energy 0.

Page 428, line 3, please read “left hand side of (14.7) when $|\xi\rangle = |\psi\rangle = 0$ is about...”

Page 432, just after (14.35). The use of the partial derivative $\partial/\partial t$ is motivated by the fact that these quantities also implicitly depend on g and that at times we will also differentiate with respect to g . Remove the sentence after (14.36) Just say “We define”

Page 434, above (11.46), in the middle of the equation it should be $\partial/\partial t$.

Page 435, line -8, $g_\varepsilon(t) := \dots$

Page 443 In the second displayed equation, there is no $n!$ in the second displayed equation.

Page 444, before the tentative LSZ.

Let us say that sets of four-momenta p_1, \dots, p_k is *indecomposable* if no strict subset of the momenta $-p_1, -p_2, p_3, \dots, p_k$ is zero.

Statement of the tentative LSZ Assume that the set of momenta p_1, p_2, \dots, p_k is indecomposable. Then...

Tentative Lemma. Assume that the sets of incoming and outgoing particles are indecomposable.

Page 445. line 9 of the proof: by our hypothesis of *indecomposability* .

Page 446. Supporting argument for the LSZ formula. Consider an indecomposable set p_1, p_2, \dots, p_k of four-momenta and set... By definition of “indecomposable, ...

Page 449, line 3, the notation A is very unfortunate. What about W ? OR \bar{A} ?

Page 449, line 10. “According to (4.70)...” This is perfectly correct, this refers to **(4.70) page 128**, but one has a tendency to read (14.70) instead of (4.70), and this is terribly confusing since (14.70) has nothing to do here.

Page 451, (14.81) it does not seem that there should be a minus sign in front of the first term. Same problem in (14.83) below.

Page 453, line -6, But according to the case $k = 1$ of Proposition 14.5.4,...

Page 454, clues to compute $K_D(p, \varepsilon)$

- To each internal vertex we assign the factor $-ig$
- For each internal vertex we enforce that the algebraic sum of all four momenta flowing on the lines adjacent to this vertex is zero by expressing one of the momentum on one of the lines as a linear combination of the others.
- We take the product of the corresponding factors.
- We integrate over all remaining momenta.

Page 455, line above (14.90) please read $\langle \Omega | \mathcal{T} \varphi(y_1) \varphi(y_2) | \Omega \rangle$.

Page 457, line 4 of 14.10, which is the interaction φ_0^3 rather than φ^3 ??

Page 458 line 9. We pretend that $\Sigma(p, 0) = \lim \dots$

Page 459. The symmetry factor 2 for the diagram has been forgotten.

Page 460 Above (14.107). ..exists. It then follows from (14.96) that ...

Page 463, equation (14.112). Of course here $p_j^2 = \mu^2$ and the limit is taken for $q_j^2 \neq \mu^2$ and $q_j^2 \rightarrow \mu^2$. In the right-hand side of the equation, the quantity $a^\dagger(p_1) a^\dagger(p_2) | 0 \rangle$ stands for an in-state “which before the interaction looks like it consists of a particle of four-momentum p_1 and a particle of four-momentum p_2 .”

Let us face it: this is not an easy result. The main difficulty is that it no longer works to be cavalier about the in and the out states and pretend as we did on page ??

that $\mathcal{H}_{\text{part}} \subset \mathcal{H}$. One has to find a way...

Page 463, line 9, ...in lemma 14.8.4 we rather inappropriately normalized...
line -6. As we explain in (14.122) below,

Page 467. two lines below (14.123) ..has a singularity at **a** smallest

Page 468, in (14.126) replace the = between p_4 and p^* by \simeq .

Page 475, line after (15.2) (but need not be integrable so that the quantities above are infinite)

Page 478 line after (15.10), Then $\deg_E P$ is the maximum possible degree in u of this polynomial as f varies. It is the largest...

Page 478, line before Lemma 15.1.13, please read “is spanned by a subset of the canonical basis”.

Page 480, missing space line above (15.14).

Page 480, middle of proof of Lemma 15.1.16. (Note that if a monomial of a given degree in $z_{m'+1}, \dots, z_m$ is present in S_{ν_1, \dots, ν_k} it gives rise by (15.13) to a monomial of the same degree in $z_{m'+1}, \dots, z_m$ in P .)

Page 481, footnote 2 It will help...

Page 482, in (15.25) replace $\leq C$ by $< \infty$.

Page 484. The quantity $\Sigma = \Sigma_v$ involved in the Feynman rules and defined on the bottom of page 390 is the sum of the momenta *leaving* the corresponding vertex v , since the momentum corresponding to a line oriented towards that vertex is given a minus sign, while the momentum of a line oriented away for that vertex is given a plus sign. So with the definition (15.30), the quantity Σ_v equals $-\mathcal{L}(x)_v + w_v$. This inconsistency has no serious consequence (besides being confusing). Indeed, the δ function satisfies $\delta^{(4)}(\Sigma) = \delta^{(4)}(-\Sigma)$ so that the formula (15.31) is correct.

Page 484 line -2, please read (15.33) instead of (15.32).

Page 486, line 6, $(R^{1,3})^\mathcal{E} \rightarrow (\mathbb{R}^{1,3})^{\mathcal{E}'}$. Line below, end ..and since $S \subset \ker \mathcal{L}$, by Lemma 15.4.3....

Page 487, two lines below (15.40). it should be better to say “It is certainly not obvious now...”

Page 489, paragraph before equation (15.42). To make matters clearer, to each vertex of the graph we attach a new edge (the “feeding edge”), connected at the other end to a source/sink of electrical current. The electrical network consists of the graph together with the feeding edges, through which electrical current can leave or enter the network.

Page 489, line -12, we attach a real number x_e ...Add a footnote Please make sure you observe that x_e is a number, not a four momentum. The reason for not choosing another notation will appear in a few line.

Page 489, replace the last lines by.

The following should be obvious **Lemma** (label lemma Alon) We have $x \in \ker \mathcal{L}$ if and only if for each $0 \leq \nu \leq 4$ we have $x^\nu \in \ker \mathcal{L}_0$.

Page 491 line -2 of the proof of 15.6.2. Write instead: But the contribution x_{e^*} of e^* occurs...Line below write: Thus $\sum_{v \in \mathcal{V}_1} \mathcal{L}_0(x)_v = \pm x_{e^*}$.

Page 492, before Th 15.7.1, I have to explain something about the use of the Euclidean structure. let us try the following.

An essential feature of the space $\ker \mathcal{L}$

Page 492. Before Th15.7.2. Replace by three lines paragraph before th 15.7.1 by the following. Let us denote by \mathcal{Q} the orthogonal complement of $(\ker \mathcal{L})^\perp$ of $\ker \mathcal{L}$ for the Euclidean structure on $(\mathbb{R}^4)^\mathcal{E}$. To understand the use of the euclidean structure here we go back to Lemma Alon. With obvious notation, a point $(y_e)_{e \in \mathcal{E}}$ belongs to \mathcal{Q} if and only if for each $0 \leq \nu \leq 3$ the point $y^\nu = (y_e^\nu)_{e \in \mathcal{E}}$ belongs to $(\ker \mathcal{L}_0)^\perp$ where \perp refers to the Euclidean structure on $\mathbb{R}^\mathcal{E}$. Thus \mathcal{Q} is also the orthogonal complement of $\ker \mathcal{L}$ when on $(\mathbb{R}^{1,3})^\mathcal{E}$ we use the natural bilinear form $(x, y) = \sum_{e \in \mathcal{E}} (x_e, y_e)$ arising from the Lorentz form on $\mathbb{R}^{1,3}$

The following simple result is fundamental for our constructions.

Before we start the proof we state the (immediate) consequence of the this result that we will use.

Rewrite the statement as follows. There exists a normalization $\mu_{\mathcal{L}}$ of the translation invariant measure on $\ker \mathcal{L}$ with the following property. Denote by w_v the sum of the external momenta leaving the vertex v . Then the integral (no coma at

the end!!) equals....

Remove the proof of the theorem.

Page 493, line -3 “It should be clear: footnote: this require a little proof that is better left as an exercise below.

Page 494, line 4: Where P is the orthogonal projection ... It seems necessary to explain well at some stage what happens there, here rather than later.

Page 494, line above (15.53) ..given $k = (k_i)_{i \leq n} \in (\mathbb{R}^{1,3})^n$ the elements... belong to $\ker \mathcal{L}$. If $x(k) = 0$ then for each $0 \leq \nu \leq 3$ we have $\sum_{i \leq n} k_i^\nu \tau_i$. Since the loops are independent, all the k_i^ν are zero, and this shows that the map $k \mapsto x(k)$ is one-to-one so that its range has dimension $4n$. When $n = \dim \ker \mathcal{L}_0$ then $4n = \dim \ker \mathcal{L}$ so that every element of $\ker \mathcal{L}$ is of the type $x(k)$

Page 489, equation (15.42). Defining $\mathcal{L}_0(x)_v$ as the amount of current *leaving* the network through the feeding edge connected to the vertex v is consistent with the definition on top of page 484 with considers the amount of current *entering* the vertex v through the adjacent edges, since these two amounts are equal by conservation of current.

Page 492 in (15.46) and in (15.47) it would be better to call the generic point of $\ker \mathcal{L}$ by z .

Page 493. It seems necessary at the begining of the chapter to recall that the series obtain diverge, that they are assymptotic series, etc.

Page 497, replace x by z in (16.1).

Page 497. Three lines above (16.2) Replace the content of the parenthesis by (In particular order to enjoy the convenient inequality (18.19) below)

Page 497. line -3. Add a footnote as to what $\mathcal{R}F$ is also defined on $(\mathbb{R}^{1,3})^\mathcal{E}$

Page 498, last equation, the middle factor should be $f(-q + (p_1 + p_2)/2)$. (Note however that this is just a formal change since $f(\ell) = f(-\ell)$.)

Page 498, footnote 1, distribution should not have a s.

Page 498, Figure 6.1, diagram (a), the flow on the edge from vertex 3 to vertex 2

should be $q - (p_2 - p - 1)/2$. There are many other possible for the flows on the edges. It would have been clearer to use the canonical flows given later, where the flow on the edge from vertex 1 to vertex 2 is $q + (2p_1 + p_2)/3$, on the edge from vertex 1 to vertex 3 is $-q + (2p_2 + p_3)/3$ and on the edge from vertex 3 to vertex 2 is $q - (p_2 - p_1)/3$. But all the possible choices give the same integral in the last line of page 498, as is seen by a translation in the space $\mathbb{R}^{1,3}$.

Page 499, formula (16.5): to understand the strange notation, think $q = k_1, \ell = k_2$.

Page 500, line -6. so that $\mathcal{L}(x)$ is the amount of momentum that enters vertex v .

Page 501, line 2 ..by how much *four-momentum enters* each vertex....

Page 501 line 6, write instead ..How much four-momentum enters the graph at each vertex. Two lines below: the point y represents a flow through the graph (=network) : y_e ...

Page 501, After first paragraph add: Going back to viewing a graph as an electrical network as on page 489, when the edges carry numbers (instead of four-momentum) canonical flows have a clean physical interpretation. Assuming as natural that each edge has the same resistance, the possible flows of current through the edges are exactly the canonical flows. This is related to the following principle. Given the amount of current which leaves or enters the network at each vertex, the actual flow of current $(x_e)_{e \in \mathcal{E}}$ minimizes the power spent in the network, power which is proportional to $\sum_{e \in \mathcal{E}} x_e^2$.

Page 501, middle .. where four momentum is allowed to leave or enter the graph....

Page 503, l 3 ..with set of edges \mathcal{E}' and set of vertices \mathcal{V}' , the set...

Page 505, just before proof of Lemma 16.5.3. $\mathcal{F}_i = \{\beta \in \mathcal{F}; \beta \subset \alpha_i\}$ and $\cup_{i \leq \bar{n}} \mathcal{F}_i = \{\beta \in \mathcal{F}; \beta \subset \alpha\}$.

Page 509, [????] bottom, discussion of diagram (a) of Figure 16.1. According to the definitions, one should use the canonical flows, not those written on the figure, for which the values are $k_{1,2} = q + (2p_1 + p_2)/3$, $k_{2,3} = -q + (p_2 - p_1)/3$, $k_{3,1} = -q - (p_1 + 2p_2)/3$. This gives the same value for $\bar{k} = (k_{1,2} + k_{2,3} + k_{3,1})$ (Please convince yourself that this has to be the case!)

Page 509, end of Exercise 16.6.3, still has divergences in the diagram β (and NOT

$\alpha \dots$) ... and despite the fact that we have taken into account all the forest contributing to renormalization (i.e. consisting only of renormalization sets).

Page 511 two lines below (16.21) ...in favor of μ , Four lines below (16.21) at small velocity where each $p_i, 1 \leq i \leq 4$ is nearly $p^* = \dots$

Page 515, Exercise 16.7.1. Given a Feynman diagram, consider the collection \mathcal{C} of internal edges such that removing this edge disconnects the diagram. Prove that the connected components of the diagram left when one removes all the edges in \mathcal{C} are $1 - PI$. Prove that the diagram obtained by contracting each of these components to a single vertex is a tree, i.e. does not contain any loop.

Page 511, two lines below (16.21) ...in favor of μ . two lines below $p_4 \simeq p^*$

Page 527 End of last paragraph: ..physicist's style, but let us stress that we present this material because it crisply brings forward the nature of the counter-term method.

Page 527, line -2, The *split* deformed theory (to be defined soon)...

Page 531, line 4 ...books: The counter terms....

Page 531, the indices are not properly placed on the counter-terms vertices at the right of Figure 17.3.

Page 533, line below (17.18):

We note that for $v \in \mathcal{V}_{\alpha_i}$ the quantity $\mathcal{L}(x)_v$ depend only on those x_e for which the edge...

Next paragraph:

Let us denote by $\tilde{\mathcal{E}}$ the set of edges of γ which are not edges of any α_i , and by $\tilde{\mathcal{V}}$ the set of vertices of γ which are not vertices of any α_i so that the cardinality of $\tilde{\mathcal{V}}$ is $n^* := n - \sum_{i \leq \tilde{n}} n_i$.

Page 537, first line of the proof of Lemma 17.8.2 $V : (\mathbb{R}^{1,3})^{\mathcal{E}'} \rightarrow \dots$

Page 533, last two lines. According to Lemma 17.8.1, it suffices to modify the Feynman rules for the deformed theory as follows (besides the obvious replacement of Δ by \square):

Page 538. middle, Remove the "truly graduate" sentence and replace by "We recall that we use a system of units...."

Page 546, line 1, versions. In (18.12) replace x by z .

Page 546, Before (8.13) As we will show later in Proposition 19.1.1, the integral... after (18.13)... where $P(k, p, \varepsilon)$ is a polynomial in ε and in the components of the p_j and the k_j ,

Page 546, remove everything from “It will clarify on line -3 until the end of Remark 18.2.3.

Page 548, middle. Consider a four vector k . Recalling (16.2),...

Page 553, Proposition 18.2.12, change the notation $R(p, \varepsilon, u)$ to $R_u(p, \varepsilon)$. Remove footnote 8, and say “the polynomial $R_u(p, 0)$ is Lorentz invariant.

Page 553, after Proposition 18.2.12,

We can now see how the proof is going to work. Keeping in mind (18.37) we see that Lemma 18.1.7 might become relevant when integrating $i p^0$ at u and p^ν for $\nu = 1, 2, 3$ fixed. But then (recalling (18.24) we have to pass the obstacle of integrating over u . As promised, we will deduce this from (18.27), basically by showing that the coefficients of the polynomial $R_u(p, \varepsilon)$ have bounded integral in u . The argument is based on the following elementary fact.

Page 544, (18.41) $R_u(p, \varepsilon)$, as well as in the proof of Lemma 18.2.15.

Page 544, Above Lemma 18.2.16 write: We need a simple fact, and state Lemma 18.2.18. Remove the rest.

Page 555. Second displayed equation after (18.45), The factor $1/a^{s-2n}$ should be inside the integration sign as it depends on p .

Page 555, line -9. It remains to prove Lorentz invariance. We prepare the proof with the following simple fact. Here move Lemma 18.2.16 and Exercise 18.2.17.

Page 555, bottom, as well of top of next page, take care of the change of notation with R_u .

Page 557 two lines above Exercise 18.3.1, a space is missing between two parenthesis.

Page 558, 559, there are some R_u to implement there.

Page 559, line -9. Replace (as is apparent...) by as is shown by the following twin of Lemma 18.2.16:

Lemma T Consider Q as in Lemma 18.2.16. Then the quantity $\sum_{0 \leq \nu \leq 3} Q(p^\nu)$ is invariant under the transformation $p \rightarrow A(p)$ when $A \in SO(4)$.

Exercise Complete the proof. Hint: same argument as Lemma 18.2.16.

Page 560, line 6 ...in which case (18.16) is proved by the change of variables $k_j \rightarrow U_z^{-1}(k_j)$ in the integral in the left-hand side. using again Lemma T.

Page 563 In this chapter we prove Theorem 16.1.1 through Zimmerman's original approach 95]. The easy part of the proof is to show that the integral (18.12) is of the type (18.15). This is the object of Proposition 19.1.1. The harder part will to prove that the integrals (18.15) are convergent through Weinberg's counting theorem. This requires grouping the terms in the forest formula in a magnificently clever way and will be done in Section 19.3. The rest is simply elaborate bookkeeping. The reader should have Chapter 16 fresh in her mind.

Given a quantity P , which is a polynomial in the variables u_1, u_2, \dots but may depend on other quantities, we denote $\deg_u P$ the degree of P in these variables. Given another quantity R which depends on these variables (but may depend on other quantities) then $R = P/Q$ is a rational function of u and we write $\deg_u R = \deg_u P - \deg_u Q$. The main result of this chapter is that given...

The proof of this will start in section [now 19.2 but will become 19.3]. In order to provide motivation for this deep result, we will first complete the deduction of Theorem 16.1.1. from Theorem 19.0.1.

Section 19.1: Preliminaries: the integral (18.12)

We prove that this integral is of the form (18.13). More precisely we have the following.

Proposition 19.1.1 Let us parameterize $\ker \mathcal{L}$ by $x = x(k) = \sum_{j \leq nk_j \tau(L_j)}$ as in Section 15.8. Then the function $\mathcal{R}F(x(k) + \Xi(p))$ takes the form of the integrand of (18.13). Furthermore the integral (18.12) takes the form (18.13)

After the statement of the proposition, remove everything until "we need first". At the end of the proof add the sentence: Finally that the integral (18.12) itself is of the type (18.13) follows from (15.45)."

Section 19.2 Proof of Theorem 16.1.1

It remains only to prove that the integral (18.14) is absolutely convergent, which we will deduce from Weinberg's power counting theorem. As p and ε are fixed, we lighten notation by not mentioning them. Let us denote by $P(k)$ and $Q(k)$ the numerators and the denominators in the integrand of (18.13) and by $Q'(k)$ the denominator in the integrand of (8.14). Our goal is to prove that for any subspace E of $(R^{1,3})^n$ we have $\deg_E P - \deg_E Q' < -\dim E$. A first observation is the $\deg_E Q = \deg_E Q'$ because both degrees equal $2N_1$ where N_1 is the number of

indices $i \leq s$ such that the quantity $\sum_{j \leq n} a_{i,j} k_j$ is not constant on E . Thus we have $\deg_E P - \deg_E Q' = \deg_E P - \deg_E Q$. Setting $N = \dim E$ and considering variables $(u_\ell)_{1 \leq \ell \leq N}$ we can find $(k_\ell)_{0 \leq \ell \leq N}$ such that $(k_\ell)_{1 \leq \ell \leq N}$ is a basis of E and $\deg_u P(k_0 + \sum_{1 \leq \ell \leq N} u_\ell k_\ell) = \deg_E P$. Since $\deg_E Q \geq \deg_u Q(k_0 + \sum_{1 \leq \ell \leq N} u_\ell k_\ell)$ we then have

$$\deg_E P - \deg_E Q \leq \deg_u P(k_0 + \sum_{1 \leq \ell \leq N} u_\ell k_\ell) - \deg_u \deg_u Q(k_0 + \sum_{1 \leq \ell \leq N} u_\ell k_\ell).$$

Next (keeping again implicit the dependence in p and ε) the quantity $\mathcal{R}F(x)$ is a rational function $A(x)/B(x)$ on $(\mathbb{R}^{1,3})^\varepsilon$ where A, B are polynomials on $(\mathbb{R}^{1,3})^\varepsilon$ and then $P(k) = A(x(k))$, $Q(k) = B(x(k))$. The desired inequality then follows from (19.1) using the sequence $f_\ell = x(k_\ell)$ (with a conflict of notation.)

Page 570, Lemma 19.3.8 The best way to understand the next four lemmas is fully grasp the meaning of the basic construction and of Figure 19.1, but we have nonetheless written a formal proof in complete detail.

Page 579, line 11, $M_\alpha(x(u))$.

Page 582, line 6, $q(q') : \mathcal{Q}' \rightarrow \mathcal{Q}$.

Page 591, The appendices that follow are of a different nature. Some deal with important matters and cannot be ignored by the reader wanting to get a deeper understanding. ... At the end, add the following. On the other hand, a mathematically inclined reader try self study of the topic in physics books will run into a very large number of mysterious looking stories. I tried to understand a few of these stories, and there is no better way to do this than to try to explain them in mathematical language. This effort being made, I saw nothing to lose by reproducing its result here, but the reader should keep in mind that these little stories are by no means necessary to follow the main ideas.

Page 597, four lines after (A.170, middle of the line, $\geq 1/2$).

Page 603, fourth line of Lemma A.4.7. $\text{card } G / \text{card } H$.

Page 603, fourth line of proof of Lemma A.4.7, $D_{w^*}^{-1} B D_{w^*}$.

Page 603, 2 lines above Lemma A.4.8. Corollary A.4.4.

Page 612, number second display equation. (zob)

Page 613 before Lemma B.1. We say that a bounded operator W commutes with A is for any $x \in \mathcal{D}(A)$ we have $W(x) \in \mathcal{D}(A)$ and $WA(x) = AW(x)$. Thus it follows from (zob) that for $x \in \mathcal{D}(A)$ we have $G(z)A(x) = zG(z)(x) = AG(z)(x)$ so that A commutes with all the operators $G(z)$ and in particular with all the operators $V(\lambda)$. Before we start our main arguments we state a general principle.

Lemma Z Consider for each n a bounded operator W_n which commutes A . If the strong limit $W = \lim_{n \rightarrow \infty} W_n$ exists it commutes with A .

The difficulty here is that $\mathcal{D}(A)$ is not norm closed. (Copy proof page 614). Then go on with Lemma B.1

Page 613 replace (B.8) by

$$\frac{d}{dt}U_\lambda(t)(x) = -U_\lambda(t)\lambda(V(\lambda) - 1)(x) = iU_\lambda(t)V(\lambda)A(x).$$

Before (B.9) ..and for $x \in \mathcal{D}(A)$ and $t > 0$ we have

$$\frac{d}{dt}U(t)(x) = iU(t)A(x). \quad (\text{zob2})$$

Moreover the operators $U(t)$ commute with A . (end of lemma)

Also, first line of Lmma B2 $\|U(t)\| \leq 1$.

Page 614 three lines below (B.12) ..derivative, and this proves (zob2). (remove the displayed equation) Finally, since the operators $V(\lambda)$ commute with A , Lemma Z shows that the operators $U_\lambda(t)$ commute with A and the operators $U(t)$ commute with A by Lemma Z again.

Page 614, replace by ...and for $x \in \mathcal{D}(A)$ and $t < 0$ we have

$$\frac{d}{dt}W(t) = iW(t)A(x).$$

Furthermore the operators $W(s)$ and $U(t)$ commute and A commutes with the operators $W(s)$.

Page 615, line 2, so that by (B.9) and (B.13) , and since the operator A commutes with the operators $U(t)$ and $W(s)$ we have $d\varphi(x)/ds = -iA\varphi(s)$. Since A is symmetric,...

Page 624 last line of proof of Proposition C.3.6. To prove this, ecalling that $P_0(f) = \tau_0(\tau_0, f)$ we compute $P_0S_0(a, b)P_0(f) = (\tau_0, f)P_0S_0(a, b)\tau_0 = \tau_0(\tau_0, f)(\tau_0, S)(a, b)\tau_0 = P_0(f)(\tau_0, S)(a, b)\tau_0$ and we use (C.36).

Page 632 Section C.5. Add a second sentence. Conversely, some of the physicists considerations may be considered as bizarre for mathematicians, and we mention a few of these below.

Page 632, line 5 after (C.74). “the function $x \mapsto \sqrt{2}x_k$. The functions of this type span the subspace \mathcal{H}_1 ... The elements of \mathcal{H}_1 can be thought as “mixtures of one-particle states”. Similarly, the (closed, linear) span \mathcal{H}_n of the elements of the type $a_{k_1}^\dagger a_{k_2}^\dagger \cdots a_{k_n}^\dagger$ represents the mixtures of the n -particle states.

Exercise Denoting by \mathcal{H}_o the space of constant functions, prove that the spaces $(\mathcal{H}_n)_{n \geq 0}$ are orthogonal and span $L^2(d\mu)$. Prove that in this manner $L^2(d\mu)$ can be viewed as the boson Fock space of \mathcal{H}_1 .

Page 640, line before (D.18) Applying the homomorphism π to this identity and using (D.17) we obtain

$$\pi(S(t)) = \exp t\pi'(X + Y)(\exp(t\pi'(X)))^{-1}(\exp(\pi'(Y)))^{-1} .$$

Replace (D.20) by

$$S(t) := (\exp(t^2[X, Y]))^{-1} \exp tX \exp tY (\exp tX)^{-1} (\exp tY)^{-1} .$$

Replace last displayed equation by

$$1 + O(t^3) = (\exp t^2\pi'([X, Y]))^{-1} \exp t\pi'(X) \dots$$

Page 645, long displayed equation in the middle of the page: $\Pi(\exp X \exp Y) = \Pi(U(1)) = \cdots \dots$

Page 648, proof of Theorem D.6.4. end. Expanding the exponential we obtain $V \exp \pi'_1(X) = \exp \pi'_2(x)V$ and since $\pi(\exp X) = \exp \pi'(X)$ we obtain that $V\pi_1(\exp X) = \pi_2(\exp X)V$ for all X in $SU(2)$. Thus $V\pi_1(X) = \pi_2(X)V$ for any $X \in SU(2)$ and V witnesses that π_1 and π_2 are equivalent.

Page 658, beginig of D.11. at the end of first sentence ...and to illustrate what physicist's students have to go through.

Page 659, line 100 from bottom. Exchange α and ν as well as β and μ in the next four lines and in the left-hand side of line -4.

Page 661, after Exercise D.12.4. It is possible to describe the irreducible contained in the tensor product of two representations (j, ℓ) (j, ℓ') in the spirit of Proposition D.7.9. Rather than producing this piece of abstraction we have felt that it would be more fun to investigate entirely “by hand” a very simple case., the tensor product of the defining representation of $SO^\uparrow(1, 3)$ with itself. To follow the computations the reader should be familiar with the technique of raising and lowering indices as explained at the end of Section 4.1. The space of tensors (x^{μ_1, μ_2}) where $0 \leq \mu_1, \mu_2 \leq 3$ is of dimension 16

Page 662, third paragraph, anti-symmetric tensors x (rather than $T...$)

Page 666, No longer number (E.6) as an equation, .. The tensor field $F_{\mu\nu}(x)$ becomes $L^\lambda_\mu L^\alpha_\nu F_{\lambda\alpha}(L^{-1}(x))$. (To understand this formula the reader is invited to compute what happens when two Lorentz transformations are applied successively and to compare with (10.6)). Here we raise and lower indexes.... transforms as $L^\mu_\lambda L^\nu_\alpha F^{\lambda\alpha}(L^{-1}(x))$.

Page 666, two lines after (E.9), $A_\nu(x) \mapsto L^\lambda_\nu A_\lambda(L^{-1}(x))$.

Page 668, last line of second paragraph, $J(\varphi)(p) = \sqrt{2\omega_p}\varphi(\mathbf{p})$. four lines from bottom, ..its Fourier transform $\varphi\hat{f}...$

Page 669, line 10, after “this is not true”. Indeed, $\mathcal{I}_t(\mathbf{x})$ is proportional to the derivative in t of the function $I(x)$ of (5.13) at $x = (ct, \mathbf{x})$.

Page 670, four lines from bottom Remove the part starting with “Therefore..” Transporting the operator X to \mathcal{H}'' using the map $f \mapsto W\hat{f}$ for \mathcal{H}'' to \mathcal{H}' we conclude that if $f \in \mathcal{S}(\mathbb{R}^3)$ is zero outside a region A then $W\hat{f} \in \mathcal{H}'$ should represent the state of a particle which is localized in A and we conclude just as before.

Page 680, Example (H.3.5). Consider the case where for some $j \leq k$ we have...

Page 682, equation after (H.19); The right hand side depends on \mathbf{x} ! There should be four times (\mathbf{x}) .

Page 684: The whole of Appendix I has to be scrapped. It is redone at the end of this erratum.

Page 692, This appendix complements Section 12.2, which the reader should review now, as well as Section 12.5. It explains one of the little stories the independent reader is bound to run into in many textbooks, but its connection with our main

story is tenuous. Our first result is actually the starting point of the rigorous study of self-adjoint operators.

Page 695, below (J.14) ..scattering states. The relevance of these states is that in a sense there are eigenvectors of H . This follows from (12.4) since $|\mathbf{p}\rangle$ is an eigenvector of H_0 (of eigenvalue $\mathbf{p}^2/(2m)$). Thus the time evolution of any element can be computed using a decomposition along this continuous basis.

Page 697, last line of second paragraph, beginning of line, “of” should be “or”

Page 707. (M11) two \circ missing.

Page 708, two lines below (M.13), two \circ missing.

Page 710, line 3 “Assuming $n \geq 1$ and thinking...One line before the end of the proof, “this holds for each $n \geq 1$...” Definition M.1.8, put the word “irreducible” in italics.

Page 714, line four of Lemma M.4.1. ...where $f'_2(x) := \dots$; line -3, two \circ missing.

Page 715, line 4, ...where $f_{2,c} := \dots$

Page 717 line 5, whenever $w \in \mathcal{S}^4$ and ...

Page 718, line 2, $\xi \geq 0$. Two lines below (M.52), extraneous s. Twice in the last two lines replace “invariant” by “Lorentz invariant”.

Page 722, line 8 \mathcal{C}_2 by a half-circle $\bar{\mathcal{C}}_2$ in the lower... line 11, and the contribution from $\bar{\mathcal{C}}_2$

Page 722, beginning of first paragraph, Considering the same contour make of \mathcal{C}_1 and \mathcal{C}_2 , but now...

Page 722, line -8, Changing x into $-x$ (or closing now the contour in the lower half plane)...

Page 740, first column, line -11, please read “momentum state space.”

The next page numbers refer to the solution of the exercises, which is online on CUPs' web site.

Page 810, Exercise 1.4.3 add $= \int dx \xi(x) \eta(x) \zeta(x)$ at the end of the first line.

Page 811, Exercise 2.5.7 at the end write ... and A^\dagger is self adjoint.

Page 812, Exercise 2.5.12. Replace the solution by the following: If g is in the domain of A^\dagger then for some constant C and all $f \in \mathcal{D}$ we have $|\int_0^1 dx g(x)^* f'(x)| \leq \|f\|_2$. For $n \in \mathbb{Z}$ let $\hat{g}(n) = \int_0^1 dx g(x)^* \exp(2i\pi nx)$ be the corresponding Fourier coefficient of g . Taking for f a finite sum $\sum_n \alpha_n \exp(2i\pi nx)$ in the previous inequality we obtain that $|\sum_n 2\pi n \alpha_n \hat{g}(n)| \leq C(\sum_n |\alpha_n|^2)^{1/2}$, so that $\sum_{n \in \mathbb{Z}} n^2 |\hat{g}(n)|^2 < \infty$ and hence $\sum_{n \in \mathbb{Z}} |\hat{g}(n)| < \infty$. Thus the Fourier series $\sum_{n \in \mathbb{Z}} \hat{g}(n) \exp(2i\pi nx)$ is uniformly convergent. Its sum is a continuous function which equals g a.e.

Page 812, Exercise 2.5.16. At the beginning of the solution add the following:

To prove that T is one-to-one consider $f \in L_2$ such that $\int_a^b dx f(x) = 0$ whenever $0 \leq a \leq b \leq 1$. Given $\varepsilon > 0$ there exists a continuous function g such that $h := f + g$ satisfies $\|h\|_2 \leq \varepsilon$ and thus $\|h\|_1 \leq \varepsilon$. Now, for $0 \leq a < b \leq 1$ we have $\int_a^b dx g(x) = \int_a^b dx h(x)$. So, if $g \geq 0$ or $g \leq 0$ on the entire interval $[a, b]$ we have $\int_a^b dx |g(x)| = |\int_a^b dx g(x)| \leq \int_a^b dx |h(x)|$. Since g is continuous, the set where $g \neq 0$ is a disjoint union of intervals $[a, b]$ such that $g \geq 0$ or $g \leq 0$ on this interval. Summing the previous inequality over these intervals shows that $\|g\|_1 \leq \varepsilon$ so that $\|f\|_1 \leq 2\varepsilon$, and since ε is arbitrary this shows that $\|f\|_1 = 0$, so that $f = 0$ a.e.

From the third sentence of the solution replace by: For f continuous and $g \in \mathcal{L}C_0$ continuous, integration by parts proves that $(T(f), g) = -(f, T(g))$. It follows by approximation that this equality still holds for $f \in L^2$ and $g \in \mathcal{L}C_0$ so that A is symmetric. The previous formula shows that $T(L^2)$ is contained in the domain of A^\dagger . But $T(L^2)$ is larger than the domain $T(\mathcal{L}_0)$ of A , so that A is not self-adjoint.

Page 814, Exercise 2.9.1. Add: In fact, the same line that precedes the statement of this exercise.

Page 814, Add the following:

Exercise 2.16.1 If φ is an approximate eigenvector for $X'(0)$ with eigenvalue a then it is an approximate eigenvector for $X'(t)$ with eigenvalue $a + pt/m$. the position has shifted from a to $a + pt/m$.

Page 814, in the solution of Exercise 2.17.2, add the following. And in case you have not already noticed, the eigenvalues of a symmetric operator are real.

Page 815, four lines from bottom, “an isometry from.” line below, “it is”. In the same exercise, replace the sentence on top of page 816 by “We proceed by induction. Consider a function f in $L^2(\mathbb{R}^n)$ which is orthogonal to each function e_{i_1}, \dots, e_{i_n} . To prove that it is zero one simply integrates in x_n first and one use the induction hypothesis.”

Page 817, line -3, there should be $\xi'(\mathbf{p})$.

Page 820, Exercise 4.5.2 Assume first **that**...

Page 821, Exercise 5.1.8, line 6: over r and $\varepsilon \geq 0$...

Page 821, Replace the nonsensical solution of Exercise 5.1.9 by the following.

We have to prove that the quantity $I_\varepsilon(x)$ given by the integral in the displayed formula below (5.16) converges uniformly on each compact set of the region we consider to an infinitely differentiable function depending only on x^2 . Assuming again \mathbf{x} to be in the direction of e_3 we compute the integral in spherical coordinates (r, θ, φ) . The volume element is $r \sin \varphi dr d\theta d\varphi$. Setting $\omega_r = \sqrt{m^2 c^2 + r^2}$ and noting that $\mathbf{x} \cdot \mathbf{p} = r|\mathbf{x}| \cos \varphi$ we may integrate in θ and φ to obtain

$$I_\varepsilon(x) = d \int_0^\infty dr \frac{r}{|\mathbf{x}| \omega_r} \exp(-i\omega_r x^0 / \hbar - \varepsilon \omega_r) \sin(|\mathbf{x}|r/\hbar),$$

where we denote by d a numerical constant that may vary between occurrences. Now, $d\omega_r/dr = r/\omega_r$ so that we may integrate by parts to obtain

$$I_\varepsilon(x) = d \int_0^\infty dr \frac{1}{x^0 + d'\varepsilon} \exp(-i\omega_r x^0 / \hbar - \varepsilon \omega_r) \cos(|\mathbf{x}|r/\hbar),$$

where d' is another constant. Consider now $b \in \mathbb{R}$ with $b^2 = x^2$ and $\text{sign } b = \text{sign } x^0$. Then there exists τ with $x^0 = b \cosh \tau$ and $|\mathbf{x}| = b \sinh \tau$. We then set $r = mc \sinh u$, so $\omega_r = mc \cosh u$ to obtain

$$I_\varepsilon(x) = d \int du \frac{\cosh u}{x^0 + d'\varepsilon} \left(\exp(-ai \cosh(u-\tau) - \varepsilon \cosh u) + \exp(-ai \cosh(u+\tau) - \varepsilon \cosh u) \right).$$

where $a = mcb/\hbar$. Therefore, using change of variable, $I_\varepsilon(x) = I_\varepsilon^1(x) + I_\varepsilon^2(x)$ where

$$I_\varepsilon^1(x) = d \int du \frac{\cosh(u+\tau)}{x^0 + d'\varepsilon} \exp(-ai \cosh u - \varepsilon \cosh(u+\tau)),$$

$$I_\varepsilon^2(x) = d \int du \frac{\cosh(u-\tau)}{x^0 + d'\varepsilon} \exp(-ai \cosh u - \varepsilon \cosh(u-\tau)).$$

The oscillatory character of the term $\exp(-ai \cosh u)$ allows by integration by parts to show that the limits as $\varepsilon \rightarrow 0$ exist with values

$$I^1(x) = d \lim_{A \rightarrow \infty} \int_{-A}^A du \frac{\cosh(u + \tau)}{x^0} \exp(-ai \cosh u),$$

$$I^2(x) = d \lim_{A \rightarrow \infty} \int_{-A}^A du \frac{\cosh(u - \tau)}{x^0} \exp(-ai \cosh u).$$

Since $\cosh(u + \tau) + \cosh(u - \tau) = \cosh u \cosh \tau$ and $x^0 = b \cosh \tau$ we finally get

$$I^1(x) + I^2(x) = \frac{d}{b} \lim_{A \rightarrow \infty} \int_{-A}^{+A} du \cosh u \exp(-ai \cosh u)$$

where $a = mcb/\hbar$. The right hand side is an infinitely differentiable function of b and hence of $\sqrt{x^2}$.

Page 825, there are mistakes in the solution of Exercise 6.9.5. (I am grateful to Jinhyun Jung for pointing this out.) Using the first displayed equation on page 825 and applying formula (4.36) we obtain

$$A\partial B/\partial t = \frac{ci}{\hbar} \iint \frac{d^3\mathbf{p}}{(2\pi\hbar)^3 2\omega_{\mathbf{p}}} \frac{d^3\mathbf{p}'}{(2\pi\hbar)^3 2\omega_{\mathbf{p}'}} p'^0 \exp(i(x, \mathbf{p} + \mathbf{p}')/\hbar) f^+(\mathbf{p}) g^+(\mathbf{p}'),$$

where $p = (\omega_{\mathbf{p}}, \mathbf{p})$ (see section 4.4) so that $p^0 = \omega_{\mathbf{p}}$, and similarly for p' . Thus

$$A\partial B/\partial t = \frac{ci}{2\hbar} \iint \frac{d^3\mathbf{p}}{(2\pi\hbar)^3 2\omega_{\mathbf{p}}} \frac{d^3\mathbf{p}'}{(2\pi\hbar)^3} \exp(i(x, \mathbf{p} + \mathbf{p}')/\hbar) f^+(\mathbf{p}) g^+(\mathbf{p}').$$

Integrating in $d^3\mathbf{x}$ and using the formula $\int d^3\mathbf{x} \exp(i(\mathbf{x}, \mathbf{p} + \mathbf{p}')/\hbar) = (2\pi\hbar)^3 \delta^{(3)}(\mathbf{p} + \mathbf{p}')$ we obtain

$$\int d^3\mathbf{x} A\partial B/\partial t = \frac{ci}{2\hbar} \iint \frac{d^3\mathbf{p}}{(2\pi\hbar)^3 2\omega_{\mathbf{p}}} d^3\mathbf{p}' \delta^{(3)}(\mathbf{p} + \mathbf{p}') \exp(ix(p^0 + p'^0)/\hbar) f^+(\mathbf{p}) g^+(\mathbf{p}').$$

For $\mathbf{p}' = -\mathbf{p}$ we have $p'^0 = p^0$ and $p' = \bar{p} := (\omega_{\mathbf{p}}, -\mathbf{p}) = (p^0, -\mathbf{p})$ so that

$$\begin{aligned} \int d^3\mathbf{x} A\partial B/\partial t &= \frac{ci}{2\hbar} \int \frac{d^3\mathbf{p}}{(2\pi\hbar)^3 2\omega_{\mathbf{p}}} \exp(2ix_0 p^0/\hbar) f^+(\mathbf{p}) g^+(\bar{p}) \\ &= \frac{ci}{2\hbar} \int d\lambda(p) \exp(2ix_0 p^0/\hbar) f^+(\mathbf{p}) g^+(\bar{p}). \end{aligned}$$

Exchanging A and B and using the transformation $p \rightarrow \bar{p}$ which preserves λ_m we find that this equals $\int d^3\mathbf{x} \partial A / \partial t B$. Proceeding in a similar fashion for the other terms one gets

$$\int d^3\mathbf{x} A \overleftrightarrow{\partial}_t B = \frac{ci}{\hbar} \int d\lambda_m(p) (f^-(p)g^+(p) - f^+(p)g^-(p)) .$$

Page 827, solution of Exercise 8.4.5, third line, the only thing we have **to** show is

Page 828, solution of Exercise 8.5.3. It is challenging to produce a beautiful geometric picture, but I realized after finishing the book that there is a way to look at this which makes the result trivial to understand (but not necessarily to visualize). The basic observation is that if for a unit vector \mathbf{v} we denote by $R_{\mathbf{v},\theta}$ the rotation of angle θ around the axis determined by \mathbf{v} , then for any unit vectors \mathbf{u}, \mathbf{v} , the loop $R_{\mathbf{v},4\pi\theta}, 1/2 \leq \theta \leq 1$ can be continuously deformed into the loop $R_{\mathbf{u},4\pi\theta}, 1/2 \leq \theta \leq 1$. This is done simply by moving continuously the axis of rotation from \mathbf{u} to \mathbf{v} . As a special case, the loop $R_{4\pi\theta}, 1/2 \leq \theta \leq 1$ can be continuously deformed into the loop $R'_{4\pi\theta}, 1/2 \leq \theta \leq 1$, where R'_θ now denote the rotation of angle θ around the third axis oriented *upside down*, which is the same as the rotation of angle $-\theta$ around the third axis, and also the same as the rotation of angle $4\pi - \theta$ around this third axis. Consequently the loop $R_{4\pi\theta}, 0 \leq \theta \leq 1$ can be deformed continuously in the loop $S_{4\pi\theta}, 0 \leq \theta \leq 1$ where S_θ is the rotation of angle θ around the third axis if $0 \leq \theta \leq 2\pi$ and is the rotation of angle $4\pi - \theta$ if $2\pi \leq \theta \leq 4\pi$ around the same axis. But it should then be obvious how to contract the loop $S_{4\pi\theta}, 0 \leq \theta \leq 1$.

Page 828 solution of Exercise 8.8.1. The reference given (that the computation is done on page 713) is nonsensical. The computation goes as follows. According to (8.23) for $A = \exp(-i\theta\sigma_3/2)$ then $\kappa(A)$ is the rotation R_θ of angle θ around the z axis, so that (8.33) shows that

$$V(\theta)(\varphi)(\mathbf{p}) = \varphi(R_\theta^{-1}(\mathbf{p})) = \varphi(p^1 \cos \theta + p^2 \sin \theta, -p^1 \sin \theta + p^2 \cos \theta, p^3)$$

and using (*), the corrected version of (8.35) this yields the formula $J_3(\varphi)(\mathbf{p}) = -i\hbar(p^2\partial_1\varphi(\mathbf{p}) - p^1\partial_2\varphi(\mathbf{p}))$.

Page 829 line 6. Two occurrences of $\kappa(A)$ should be $\kappa(A^{-1})$. Furthermore, the statement of Exercise 8.8.2 is wrong, the wave function is multiplied by a phase $\exp(i\theta(j/2 - k))$ (as the proof shows), not $\exp(-i\theta(j/2 - k))$. However, the statement of Exercise 8.8.2 becomes correct when we amend (8.35) by adding a minus sign as explained above.

Page 830, last line. replace “I see..” by: There are exactly two possible maps which achieve this, namely...”. To see this we recall Schur’s lemma, which implies that when two linear maps W, W' witness that two irreducible representations are equivalent we have $W = \lambda W'$ for $\lambda \in \mathbb{C}$. And here the condition that $S(P')^2$ be the identity forces that $\lambda = \pm 1$.

Page 831, line -1, there is a $d\lambda$ which should be $d\lambda_m$.

Page 832, Exercise 9.6.8. Replace (d) by the following. The map $z \mapsto W(z) := (p^1 + ip^2)/(p^0 + p^3)$ provides an identification of \mathbb{C} by the quotient of the equivalence relation $p\mathcal{R}p'$ iff $p' = \lambda p$ for some $\lambda > 0$. The action of $SL(2, \mathbb{C})$ on $X_0 \setminus \{0\}$ respects this equivalence relation. Furthermore, $W(p)$ is the unique complex number such that $M(p) = \alpha ZZ^\dagger$ where $\alpha \in \mathbb{R}$ and Z is the column matrix $\begin{pmatrix} 1 \\ W(p) \end{pmatrix}$. The relation $M(A(p)) = AM(p)A^\dagger = \alpha AZ(AZ)^\dagger$ implies that the quotient action of $SL(2, \mathbb{C})$ on \mathbb{C} is the one we study here, and (9.46)....

Page 833, line 6. $\xi(A, W(p))$

Page 833, Ex 9.6.11 second line g'_{n_1, \dots, n_j} ; line 4, by.

Page 833 last line of exercise 9.8.14. norm (9.48), see...

Page 835 line 3, Exercise 4.1.5.

Page 836, line before Exercise 10.5.3 : formula (10.22).

Page 837, Exercise 10.7.2., Write instead: The quantities $(u(\mathbf{p}, q)_k)$ for $1 \leq k \leq N...$

Page 839, fourth line of Exercise 11.1.2, there is a) missing.

Page 839, end of Exercise 11.1.3, add the following. We had taken $g = 1$. Putting it back, as $g \rightarrow 0$ the term $g(z, H_I(z))$ is of order g^3 , and $\|v_0 + z\|^2 = \|v_0\|^2 + \|z\|^2$ and dropping the term $\|z\|^2$ makes only a change of order g^3 .

Page 847, Exercise 16.7.1. (a) Consider an edge e of such a component A so that $e \notin \mathcal{C}$ and therefore e is an edge of a loop of the original diagram. None of the edges of this loop belongs to \mathcal{C} so that it is entirely contained in A and thus removing e does not disconnect A . (b) It follows from (a) that a loop in the contracted diagram arises from a loop in the original diagram, so that it cannot contain any edge in \mathcal{C} .

Page 855, Exercise D.1.11, there is an extraneous X on the second line.

Page 857, Exercise D.12.4. Replace the solution by the following:

We first complete the proof of Exercise 8.4.4. Let us consider the map $M : x \mapsto M(x)$ from \mathbb{C}^4 to \mathcal{M} given by the formula (8.19). Then by definition of κ for each $x \in \mathbb{R}^4$ we have $M(\kappa(A)(x)) = \theta(A)(M(x))$ so that by linearity this formula also holds for $x \in \mathbb{C}^4$. Thus $M\kappa(A) = \theta(A)M$ and κ (seen as a representation of $SL(2, \mathbb{C})$ on \mathbb{C}^4) is indeed equivalent to the representation $(1, 1)$ of $SL(2, \mathbb{C})$. The rest is obvious.

New version of Appendix I

We assume here that the reader is familiar with Section E.2. Ideally, one would like the components $A_\nu(x)$ of the electromagnetic four potential. In Section 10.22 we saw that the most natural approach fails. The “standard approach” would be finding a Lagrangian that describes the correct equations of motion, imposing canonical commutation relations between the “dynamic variables” and their conjugates. One can look e.g. in Schewber’s book [75, Chapter 9, section b] for such a trial and why it fails.

Physicists have developed an intriguing method to deal with these problems. the called Gupta-Bleuler formalism. It is not so obvious to understand what they mean by statements such as “the Hilbert space of the photon is endowed with an indefinite metric.” A concise treatment of this approach can be found in the book of Dimock [23].

In the present section we show that the objectives of this program can be achieved in a very simple manner without resorting to any disturbing concepts. (I apologize for having overlooked this in the first edition of this book.) The key idea is the approach of Section 9.13, which we review now. We recall the sesqui-linear form on \mathbb{C}^4 given by the formula (9.93):

$$(x, y)_\eta = -x^{0*}y^0 + \sum_{i \leq 3} x^{i*}y^i.$$

Given $p \in X_0$ we recall the space

$$\mathcal{V}_p = \{x \in \mathbb{C}^4 ; (x, p)_\eta = 0\}.$$

Consider the space \mathcal{H}_0 of functions φ from X_0 to \mathbb{C}^4 for which $(\varphi, \varphi)_\eta < \infty$ where for two such functions φ, ψ we define their inner product by

$$(\varphi, \psi)_\eta = \int d\lambda_m(p) (\varphi(p), \psi(p))_\eta.$$

As we have seen, this inner product is positive, but not definite positive. We denote by $\mathcal{H}_{\text{null}}$ the subspace of \mathcal{H}_0 consisting of the elements φ for which $(\varphi, \varphi)_\eta = 0$ and

by $\mathcal{H}_{\text{phys}}$ the quotient $\mathcal{H}_0/\mathcal{H}_{\text{null}}$ which is now a Hilbert space. Proposition 9.13.1. shows that there is a unitary representation $U(c, C)$ of \mathcal{P}^{*+} on $\mathcal{H}_{\text{null}}$ which makes this space appear as “the natural Hilbert space to describe a single photon”. Let us denote the corresponding Boson Fock space by $\mathcal{B}_{\text{photon}}$ and by $U_{\mathcal{B}}(c, C)$ the natural extension of $U(c, C)$ to $\mathcal{B}_{\text{photon}}$. Thus elements of $\mathcal{B}_{\text{photon}}$ represent multiple photons. Therefore it is very natural that the quantization of anything having to do with the electromagnetic field *be done by operators on $\mathcal{B}_{\text{photon}}$* .

For $\psi \in \mathcal{H}_{\text{phys}}$ we denote by $B(\psi)$ and $B^\dagger(\psi)$ the corresponding annihilation and creation operators on $\mathcal{B}_{\text{photon}}$. Considering four functions $f^\nu, 0 \leq \nu \leq 3$ in $\mathcal{S}_{\mathbb{R}}^4$ and assume that for each $p \in X_0$ we have $\varphi(p) := (\widehat{f^0}(p), \widehat{f^1}(p), \widehat{f^2}(p), \widehat{f^3}(p)) \in \mathcal{V}_p$, that is $(\varphi(p), p)_\eta = 0$, a condition that we write simply $p_\nu \widehat{f^\nu}(p) = 0$. Then the function φ on X_0 belongs to \mathcal{H}_0 . We denote by $[\varphi]$ its image in $\mathcal{H}_{\text{phys}}$ and we define the operator $A_\nu(f^\nu)$ as

$$A_\nu(f^\nu) = B^\dagger([\varphi]) + B([\varphi]). \quad (\textit{titi})$$

The idea for this notation is as follows. If we could define for each ν and $f \in \mathcal{S}_{\mathbb{R}}^4$ the operator $A_\nu(f)$ in the proper way, then the sum $A_\nu(f^\nu)$ would be given by (titi). Unfortunately we cannot do that, we don't know how to define $A_\nu(f)$, but nonetheless, under proper conditions we can define this sum. Note also that in (titi) there should be a multiplicative constant, if only to get the proper dimension, but we do not care about this now.

Consider now the case where for some function $f \in \mathcal{S}_{\mathbb{R}}^4$ we have $f^\nu = \partial^\nu f$. Here we remember that we think of f as a function on $\mathbb{R}^{1,3}$ and that $\partial^\mu f := \partial f / \partial x_\mu$ so that $\widehat{\partial^\nu f} = -ip^\nu \widehat{f}(p)$. That is $\varphi(p)$ is proportional to p so that $[\varphi] = 0$ and thus by (titi) we have $A_\nu(\partial^\nu f) = 0$. If it were true that the operators A_ν were defined individually, we would have by definition of the derivative of a distribution that $\partial^\nu A_\nu(f) = -A_\nu(\partial^\nu(f))$. Thus we can interpret the fact that $A_\nu(\partial^\nu f) = 0$ by saying “we are working in the Lorenz gauge and $\partial^\nu A_\nu = 0$ ”.

The four-potential is not observable, so it is not a real nuisance that we cannot quantize it. This potential is just a tool, the physical quantities of interest are the components $F_{\mu,\nu}$ of the electromagnetic tensor. Consider $f \in \mathcal{S}_{\mathbb{R}}^4$ and for $0 \leq \eta \leq 3$ define f^η as follows: if $\eta \notin \{\mu, \nu\}$ then $f^\eta = 0$. Otherwise, $f^\mu = \partial_\nu f$, $f^\nu = -\partial_\mu f$. We then define

$$F_{\mu\nu}(f) = A_\eta(f^\eta),$$

where the right hand side is given by (titi). As a sanity check we should verify what happen under the action of the Poincaré group. For $C \in SL(2, \mathbb{C})$ we denote again by C the matrix of $\kappa(C)$.

Proposition For a function $f \in \mathcal{S}_{\mathbb{R}}^4$ we have

$$U_{\mathcal{B}}(c, C) \circ F_{\mu\nu}(f) \circ U_{\mathcal{B}}(c, C)^{-1} = C^\lambda{}_\mu C^\alpha{}_\nu F_{\lambda\alpha}(V(c, C)f),$$

where $V(c, C)(f)(x) = f(C^{-1}(x - c))$.

The proof is obtained by a straightforward calculation, which exemplifies the fact that straightforward and easy are two different measures!