

Create convexity in 3 (or 100?) steps only!

Consider an integer N . Let us say that a compact subset A of \mathbf{R}^N is **balanced** if

$$x \in A, \lambda \in \mathbf{R}, |\lambda| \leq 1 \Rightarrow \lambda x \in A .$$

Let us denote by γ_N the canonical Gaussian measure on \mathbf{R}^N .

Problem. Prove that there exists an integer q , such that for all N and every compact balanced set A of \mathbf{R}^N such that $\gamma_n(A) \geq 1/2$, one can find a **convex** compact set $C \subset A + \cdots + A$ (with q terms on the right) such that $\gamma_n(C) \geq 1/2$.

In words: **finitely many steps, independently of dimension, suffice to create convexity.**

Even if the constant L is large one cannot always find such a C inside $L(A + A)$, but I do not know if one can find it inside $L(A + A + A)$.

This sounds to me as a fundamental question. The silence of the convexity specialists about it has been so far deafening.

This problem is discussed in my paper “Are all sets of positive measure essentially convex?” Operator theory: Advances and applications. Vol. 77, Birkhäuser, 1995, p. 295-310. It is somewhat related to the “combinatorics problem” but you will get US \$ 2000 for solving both.